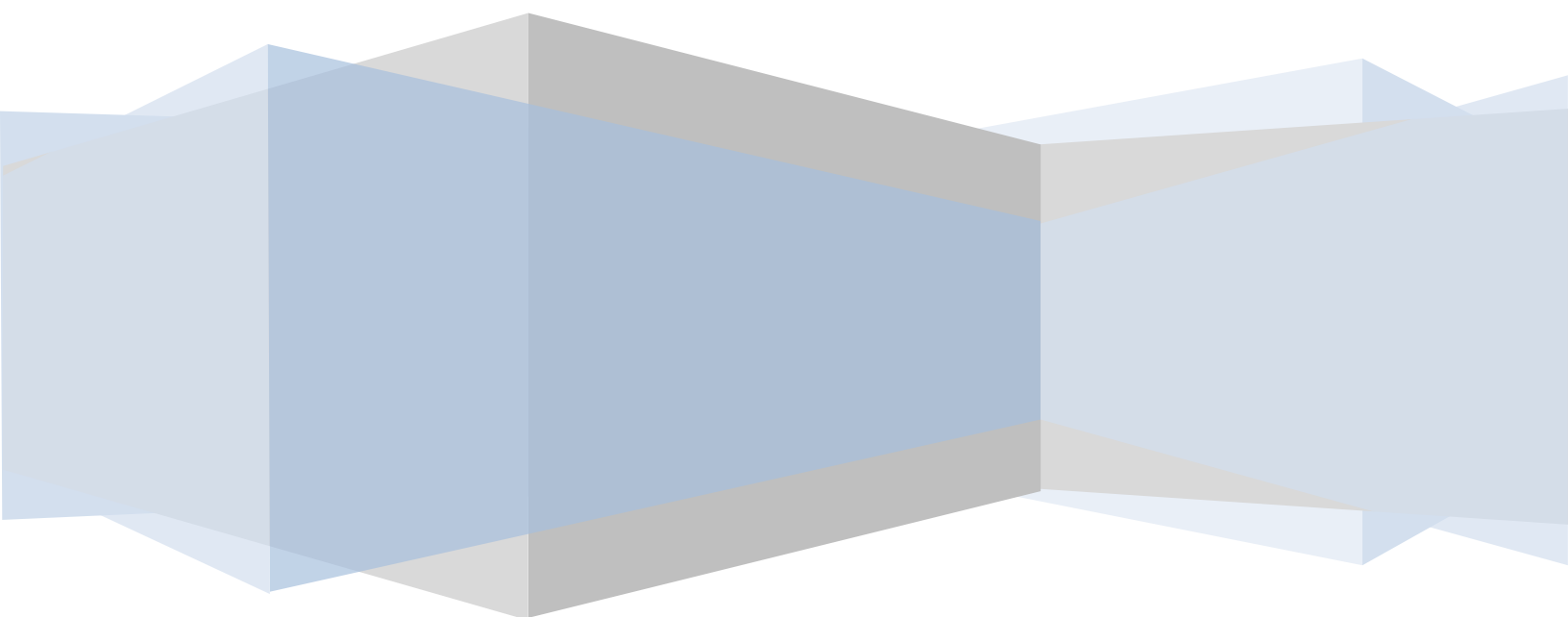


# **Metric Math for Wastewater Operators**

## **A Guide to Using the ABC and EOCP Canadian Formulas**





## Table of Contents

Introduction.....	6
The Metric System.....	7
Units and Equivalents in the Metric System .....	8
ppm and mg/L .....	8
Significant Figures and Rounding .....	9
Zero – Is it significant or not? .....	10
Basic Math Skills .....	10
Order of Operation – BEDMAS .....	10
Addition and Subtraction .....	11
Multiplication and Division.....	11
Arithmetic Mean, Median, Range, Mode, and Geometric Mean .....	12
Percent Calculations.....	14
Exponents and Powers of 10.....	16
Geometry – Perimeter and Circumference, Area and Volume .....	18
Pi ( $\pi$ ) .....	19
The constant 0.785.....	20
Perimeter and Circumference .....	20
Volume .....	25
The Megaliter Shortcut.....	28
Things That Are Equal to One.....	29
Set up of Calculations in this Manual .....	29
Before we get started.....	30
PART 1 – FORMULAS AND PRACTICE QUESTIONS – EOCP HANDOUT .....	31
Solids (Mass) Calculation.....	31
Percent (%) Removal Calculation .....	32
% Removal - Volatile Solids Destruction / Moisture Reduction .....	32
Flow and Velocity Calculations.....	33
Detention Time (or Hydraulic Retention Time) Calculations .....	35
Hydraulic Loading Rate Calculations .....	36
Values obtained from these equations are often expressed as:.....	36
Aeration basins, Rotating biological contactors, Filter flow and filter backwash .....	36

## Metric Mathematics for Operators

Litres per Capita per day .....	38
Hydraulic Overflow Rate.....	38
Surface Overflow Rate (SOR) Calculations .....	38
Weir Overflow Rate (WOR) Calculations .....	39
Values obtained from these equations are often expressed as:.....	39
Chemical Feed Rate / Chlorine Feed Rate .....	40
Chlorine Dosage.....	40
Chlorine dose, chlorine demand, chlorine residual calculations.....	41
Chemical Feed Rate Calculations.....	42
Ratio Calculations .....	44
Organic Loading Rate.....	45
Aeration Basins, Rotating Biological Contactors, Trickling Filters, Clarifiers.....	45
Wastewater Sludge Calculations (Activated Sludge ).....	46
Sludge Volume Index and Sludge Density Index Calculations.....	46
Sludge Volume Index (SVI).....	47
Sludge Density Index (SDI).....	47
Food to Microorganism (F:M) Ratio Calculations.....	48
Sludge Recycle Rate.....	49
Sludge Return Rate, % .....	50
Sludge Wasting Rate.....	50
Mean Cell Residence Time /Sludge Age / Solids Retention Time .....	52
Mean Cell Residence Time (MCRT) .....	52
Solids Retention Time (SRT) .....	52
Sludge Age .....	53
Solids under Aeration .....	54
Waste Activated Sludge (WAS) Pumping Rate .....	54
Slope and Grade .....	55
Pumping Calculations .....	55
Power Calculations .....	56
Efficiency Calculations .....	57
PART 2 – FORMULAS AND PRACTICE QUESTIONS – ABC HANDOUT .....	59
Basic Electricity.....	59

Metric Mathematics for Operators

Biochemical Oxygen Demand (BOD<sub>5</sub>) – Unseeded and Seeded..... 60

Chemical Feed Pump Setting - % Stroke ..... 61

Composite Sample..... 61

Cycle Time – Pumping ..... 62

Filtration Rate Calculations ..... 62

    Filter Flow and Filter Backwash Rate..... 63

    Filter Backwash Rise Rate..... 63

    Vacuum Coil Filter Yield..... 63

Force and Pressure ..... 64

Basic Chemistry ..... 65

    Number of Moles..... 65

    Number of Equivalent Weights ..... 66

    Molarity ..... 67

    Normality..... 67

    Alkalinity..... 68

    Hardness..... 68

    Oxygen Uptake Rate (OUR) ..... 68

    Specific Oxygen Uptake Rate (SOUR) ..... 69

Solids Concentration ..... 70

Population Equivalent Calculations..... 70

Recirculation Ratio – Trickling Filter..... 71

Density and Specific Gravity..... 72

    Density..... 72

    Specific Gravity of Liquids..... 72

    Specific Gravity of Solids..... 73

Typical Certification Questions for Level 1..... 74

Typical Certification Questions for Level 2..... 75

Answer key for practice questions ..... 77

Appendix 1 – EOCP Formula Sheets ..... 78

Appendix 2 – ABC Formula Sheets ..... 82

## Introduction

This manual was written to provide operators of wastewater treatment plants with a guide to the use of the formulas found in the handouts provided to certification examination candidates. The formulas used will be those found in the Canadian version of the Association of Boards of Certification's (ABC) handout and in the handout provided by the Environmental Operators Certification Program (EOCP). Both handouts are included as Appendices

Each formula is accompanied by one or more solved examples of a question which would require the use of the formula to obtain a solution. Each of the sample questions begins with **the question stated in bold text**. Each of the sample questions contains the basic equation used, a step by step guide to developing the information needed to solve the question and the solved question using a "dimensional analysis" approach which first sets out the question in words and then solves it by substituting the appropriate numerical value. Many of the questions will have application to other disciplines. For example, operators in any of the four disciplines may need to calculate hydraulic detention time – it may be called a reservoir for a water distribution operator, a wet well for a collection system operator, and a clarifier for either a wastewater or water treatment plant operator but the basic mathematical concept is the same.

Additional information can be found in the publications of the following organizations and agencies:

American Water Works Association  
Association of Boards of Certification  
California State University, Sacramento  
Environment Canada  
Metcalf and Eddy / AECOM  
Provincial and State Operator Certification Programs  
United States Environmental Protection Agency (USEPA)  
Water Environment Federation

Graeme Faris

January 2018

Copyright © 2018 Graeme Faris

All parts of this publication may be reproduced in any form, by any photographic, mechanical or other means, or used in any information storage and retrieval system, without the written permission of the author provided that it is used to advance operator education and training and made available to the end user at no cost.

## The Metric System

The metric system is used in all of the industrialized countries of the world except the United States.

Canada began the conversion to a metric system of measurement in 1970 and by 1975 it was in universal use throughout the country.

Introduced in France in 1779 the metric system originally was limited to two units – the metre and the kilogram.

The metre (meter in the US), symbol m, is the base unit of length in the International System of Units (SI). Originally intended to be one ten-millionth of the distance from the Earth's equator to the North Pole (at sea level), since 1983, it has been defined as "the length of the path travelled by light in vacuum during a time interval of  $1/299,792,458$  of a second ( $\approx 3 \times 10^{-9}$  seconds).

The kilogram, also known as the kilo, symbol kg, is the base unit of mass in the International System of Units and is defined as being equal to the mass of the International Prototype Kilogram (IPK). The IPK is made of a platinum-iridium alloy, which is 90% platinum and 10% iridium (by mass) and is machined into a cylinder with a height and diameter of approximately 39 millimeters to minimize its surface area. The cylinder has a mass which is almost exactly equal to the mass of one liter of water.

The metric system is decimal, except where the non-SI units for time (hours, minutes, seconds) and plane angle measurement (degrees, minutes, seconds) are concerned. All multiples and divisions of the decimal units are factors of the power of ten.

Decimal prefixes are a characteristic of the metric system; the use of base 10 arithmetic aids in unit conversion. Differences in expressing units are simply a matter of shifting the decimal point or changing an exponent; for example, the speed of light may be expressed as 299,792,458 m/s or  $2.99792458 \times 10^8$  m/s.

A common set of decimal-based prefixes is applied to some units which are too large or too small for practical use without adjustment. The effect of the prefixes is to multiply or divide the unit by a factor of ten, one hundred or an integer power of one thousand. The prefix *kilo*, for example, is used to multiply the unit by 1000, and the prefix *milli* is to indicate a one-thousandth part of the unit. Thus, the *kilogram* and *kilometre* are a thousand grams and metres respectively, and a *milligram* and *millimetre* are one thousandth of a gram and metre respectively. These relations can be written symbolically as:

$$1 \text{ mg} = 0.001 \text{ g} \qquad 1 \text{ km} = 1000 \text{ m}$$

When applying prefixes to derived units of area and volume that are expressed in terms of units of length squared or cubed, the square and cube operators are applied to the unit of length including the prefix, as illustrated here:

$$1 \text{ mm}^2 \text{ (square millimetre)} = (1 \text{ mm})^2 = (0.001 \text{ m})^2 = 0.000 \text{ 001 m}^2$$

$$1 \text{ km}^2 \text{ (square kilometre)} = (1 \text{ km})^2 = (1000 \text{ m})^2 = 1,000,000 \text{ m}^2$$

$$1 \text{ mm}^3 \text{ (cubic millimetre)} = (1 \text{ mm})^3 = (0.001 \text{ m})^3 = 0.000 \text{ 000 001 m}^3$$

$$1 \text{ km}^3 \text{ (cubic kilometre)} = (1 \text{ km})^3 = (1000 \text{ m})^3 = 1,000,000,000 \text{ m}^3$$

On the other hand, prefixes are used for multiples of the non-SI unit of volume, the litre (L), or the stere (cubic metre). Examples:

$$1 \text{ mL} = 0.001 \text{ L} \qquad 1 \text{ kL} = 1,000 \text{ L} = 1\text{m}^3$$

The tonne (1,000 kg), the litre (now defined as exactly 0.001 m<sup>3</sup>), and the hectare (10,000 m<sup>2</sup>), continue to be used alongside the SI units.

### Units and Equivalents in the Metric System

Tera	(T)	10 <sup>12</sup>	1,000,000,000,000
Giga	(G)	10 <sup>9</sup>	1,000,000,000
Mega	(M)	10 <sup>6</sup>	1,000,000
Kilo	(K)	10 <sup>3</sup>	1,000
Hecto	(H)	10 <sup>2</sup>	100
Deca	(D)	10 <sup>1</sup>	10
Deci	(d)	10 <sup>-1</sup>	1/10
Centi	(c)	10 <sup>-2</sup>	1/100
Milli	(m)	10 <sup>-3</sup>	1/1,000
Micro	(μ)	10 <sup>-6</sup>	1/1,000,000
Nano	(n)	10 <sup>-9</sup>	1/1,000,000,000
Pico	(p)	10 <sup>-12</sup>	1/1,000,000,000,000

### ppm and mg/L

The acronym ppm stands for parts per million and was commonly used in the pre-metric era. In the metric system we use the acronym mg/L which stands for milligram per litre. The metric system assigns a weight of one kilogram to one litre of water. One kilogram of water contains one million milligrams and thus a value of one milligram per litre is exactly equivalent to one part per million parts. The two terms can be used interchangeably.

Proof:

Consider that by definition, 1 Litre of water weighs 1 kilogram

1 kilogram contains 1,000 grams

1 gram contains 1,000 milligrams

Therefore, 1 kilogram contains 1,000 grams × 1,000 milligrams/gram = 1,000,000 milligrams (mg)

Thus,

$$\frac{1 \text{ mg}}{\text{L}} = \frac{1 \text{ mg}}{\text{kg}} = \frac{1 \text{ mg}}{1,000 \text{ g}} = \frac{1 \text{ mg}}{1,000,000\text{mg}} = 1 \text{ part/million parts} = 1\text{ppm}$$



Quantity Measured	Unit	Symbol	Relationships
Distance, length, width, thickness, girth, etc.	millimetre centimetre metre kilometre	mm cm m km	10 mm = 1 cm 100 cm = 1 m 1,000 m = 1 km
Mass (Weight)	milligram gram kilogram tonne	mg g kg t	1,000 mg = 1 g 1,000 g = 1 kg 1,000 kg = 1 t
Area	square metre hectare square kilometre	m <sup>2</sup> ha km <sup>2</sup>	10,000 m <sup>2</sup> = 1 ha 100 ha = 1 km <sup>2</sup>
Volume	millilitre cubic centimetre litre cubic metre Megalitre	mL cm <sup>3</sup> (or cc) L m <sup>3</sup> ML	1 cm <sup>3</sup> = 1 mL 1,000 mL = 1 L 1,000L = 1 m <sup>3</sup> 1,000 m <sup>3</sup> = 1 ML
Velocity (Speed)	metres/second kilometre/hour	m/s km/h	
Temperature	degree Celsius	°C	
Pressure	kilopascal	kPa	9.8 kPa = 1 m of head
Energy	joules kilowatt-hour	J kWh	
Power	watt	W	

## Significant Figures and Rounding

When we use a handheld calculator or the calculator function on our smart phone, laptop or tablet it is not uncommon to get an answer to the fifth or sixth decimal point. But is that answer accurate? Is that level of precision necessary? The accuracy of any answer is based on the accuracy of the values used in determining the answer and that depends on the precision of the measuring instrument or even the skill of the person using the instrument.

The rules for determining the number of significant figures or digits that an answer should have are relatively straightforward.

There are three rules that apply to “rounding off” numbers until the appropriate numbers of significant figures remain:

1. When a figure less than five is dropped, the next figure to the left remains unchanged. For example, the number 11.24 becomes 11.2 when it is required that the four be dropped
2. When the figure is greater than five that number is dropped and the number to the left is increased by one. For example, 11.26 becomes 11.3

The third rule, which is less commonly used, helps to prevent rounding bias in long series of numbers.

- When the figure that needs to be dropped is a five, round to the nearest even number. For example, 11.35 becomes 11.4 and 46.25 becomes 46.2

### Zero – Is it significant or not?

A zero may be a significant figure, if it is a measured value, or be insignificant and serve only as a place holder or spacer for locating the decimal point. If a zero or zeroes are used to give position value to the significant figures in the number, then the zero or zeroes are not significant. Consider this:

$$1.23 \text{ mm} = 0.123 \text{ cm} = 0.000123 \text{ m} = 0.00000123 \text{ km}$$

In the example above, the zeroes are insignificant and only give the significant figures, 123, a position that indicates their value.

## Basic Math Skills

### Order of Operation – BEDMAS

BEDMAS is an acronym which can be used to help remember the correct order in which mathematical operations are carried out when solving an equation. That order is:

1	Brackets	( )
2	Exponents	$3^2$
3	Division	$\div$
4	Multiplication	$\times$
5	Addition	$+$
6	Subtraction	$-$

**Example 1 – Consider the equation:  $(5 - 2)^2 + 4(2 + 1)/6 - 1$**

Solve using BEDMAS

$$\text{Brackets } (5 - 2)^2 + \frac{4(2 + 1)}{6} - 1 = (3)^2 + \frac{4(3)}{6} - 1$$

$$\text{Exponents } (3)^2 + \frac{4(3)}{6} - 1 = 9 + \frac{4(3)}{6} - 1$$

$$\text{Division / Multiplication } 9 + \frac{4(3)}{6} - 1 = 9 + \frac{12}{6} - 1 = 9 + 2 - 1$$

$$\text{Addition } 9 + 2 - 1 = 11 - 1$$

$$\text{Subtraction } 11 - 1 = 10$$

When solving a fractional expression, you treat each part (the numerator and the denominator) as separate equations and apply the rules of BEDMAS accordingly. Finally, divide the numerator by the denominator.

**Example 2 – Consider the equation:  $8 + 3^2(3 \times 5) - 6(3 + 5)$**

Brackets  $8 + 3^2(3 \times 5) - 6(3 + 5) = 8 + 3^2(15) - 6(8)$

Exponents  $8 + 3^2(15) - 6(8) = 8 + 9(15) - 6(8)$

Division / Multiplication  $8 + 9(15) - 6(8) = 8 + 135 - 48$

Addition  $8 + 135 - 48 = 143 - 48$

Subtraction  $143 - 48 = 95$

### Addition and Subtraction

In addition, and subtraction, only similar units expressed to the same number of decimal places may be added or subtracted. The number with the least number of decimal places a limit on the number of decimals that the answer can justifiably contain. For example, suppose you have been asked to add together the following values: 446 mm + 185.22 cm + 18.9 m. First convert the quantities to similar units (in this case metres) and then chose the least accurate number, which is 18.9. As it only has one digit to the right of the decimal point, the other two values will have to rounded off.

$$\begin{array}{rcl} 446 \text{ mm} & = & 0.446 \text{ m} = 0.4 \text{ m} \\ 185.22 \text{ cm} & = & 1.8522 \text{ m} = 1.8 \text{ m} \\ 18.9 \text{ m} & = & 18.9 \text{ m} = \underline{18.9 \text{ m}} \\ & & 21.1 \text{ m} \end{array}$$

When adding numbers (including negative numbers), the rule is that the least accurate number will determine the number reported as the sum. The number of significant figures reported in the sum cannot be greater than the least significant figure in the group being added. In the next example, the least precise number, 170, dictates that the other three numbers will have to be changed (rounded off) before addition is done.

$$\begin{array}{rcl} 1.023 \text{ g} & = & 1 \text{ g} \\ 23.22 \text{ g} & = & 23 \text{ g} \\ 170 \text{ g} & = & 170 \text{ g} \\ 1.008 \text{ g} & = & \underline{1 \text{ g}} \\ & & 195 \text{ g} \end{array}$$

### Multiplication and Division

The rules for rounding off in multiplication and division are different for those used in addition and subtraction. In multiplication and division, the number with the fewest significant figures will dictate how the answer is finally written. Suppose we have to multiply 26.56 by 6.2.

$$(26.56)(6.2) = 164.672$$

In the equation above, the first number has four significant figures while the second number only has two. Therefore, the answer should only be written with two significant figures as 160 because the least precise value (6.2) only has two significant figures.

**Arithmetic Mean, Median, Range, Mode, and Geometric Mean**

The term “arithmetic mean” is just another way of saying “average”.

The arithmetic average of a series of numbers is simply the sum of the numbers divided by the number of values in the series.

The equation is:

$$\text{Average} = \frac{\text{Sum of all terms}}{\text{Number of terms}}$$

**What is the average concentration of volatile acids in a digester supernatant given the following data? All values given are in mg/L**

Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Sun
234	261	280	272	259	257	244

Insert known values and solve

$$\text{Average VSS} = \frac{234 + 261 + 280 + 272 + 259 + 257 + 244}{7 \text{ days}} = 258 \text{ mg/L}$$

**Calculate the 7 day running average for BOD<sub>5</sub> removal during days 8, 9 and 10 given the following data:**

Day	BOD <sub>5</sub> , mg/L	Day	BOD <sub>5</sub> , mg/L
1	212	9	226
2	231	10	211
3	244	11	245
4	235	12	206
5	217	13	193
6	202	14	188
7	194	15	189
8	209	16	204

Step 1 – Calculate the average for day 8 and the previous 6 days (7 days total)

$$\text{Average, days 2 to 8} = \frac{231 + 244 + 235 + 217 + 202 + 194 + 209}{7} = 219 \text{ mg/L}$$

## Metric Mathematics for Operators

Step 2 – Calculate Day 9, 7 day running average by dropping day 2 and adding day 9

$$\text{Average, days 3 to 9} = \frac{244 + 235 + 217 + 202 + 194 + 209 + 226}{7} = 218 \text{ mg/L}$$

Step 3 - Calculate Day 10, 7 day running average by dropping day 3 and adding day 10

$$\text{Average, days 4 to 10} = \frac{235 + 217 + 202 + 194 + 209 + 226 + 211}{7} = 213 \text{ mg/L}$$

**Given the following data, calculate the unknown values**

Day	Effluent BOD <sub>5</sub> , mg/L	Unknown values
Monday	28	<b>Arithmetic mean, mg/L</b>
Tuesday	32	<b>Median, mg/L</b>
Wednesday	34	<b>Range, mg/L</b>
Thursday	32	<b>Mode, mg/L</b>
Friday	29	<b>Geometric mean, mg/L</b>
Saturday	23	
Sunday	35	

*Note: a Scientific calculator is required to determine the geometric mean*

**Calculate the arithmetic mean (average)**

$$\text{Arithmetic mean} = \frac{28 + 32 + 34 + 32 + 29 + 23 + 35}{7} = 30.4, \text{ round to } 30 \text{ mg/L BOD}_5$$

**Determine the median of BOD<sub>5</sub> mg/L**

To determine the median value, put the data in ascending order and choose the middle value

1	2	3	4	5	6	7
23	28	29	32	32	34	35

In this case, the middle or median value is 32 mg/L BOD<sub>5</sub>

**Determine the mode of BOD<sub>5</sub> mg/L**

Mode is the measurement that occurs most frequently. In this case it is 32 mg/L as it appears twice in the data set.

**Determine the range of BOD<sub>5</sub> mg/L**

The equation is: Range = Largest value – smallest value = 35 mg/L – 23 mg/L = 12 mg/L BOD<sub>5</sub>

### Determine the geometric mean

The equation is: Geometric mean =  $[(x_1)(x_2)(x_3)(x_4)(x_5)(x_6)(x_7)]^{1/n}$

Where x = the value of the measurement and n = the number of measurements.

Insert known values and solve

$$\text{Geometric mean} = (23 \times 28 \times 29 \times 32 \times 32 \times 34 \times 35)^{1/7} = (22,757,826,560)^{1/7} = 30.2 \text{ mg/L BOD}_5$$

*Note: for any series of numbers, the geometric mean will always be less than the arithmetic mean.*

*Determination of the geometric mean requires a scientific calculator with an nth root function. Current EOCB practice is to only include mathematical questions that can be solved with a basic four function calculator so it is unlikely that a question involving solving for the geometric mean will appear on a certification exam.*

### Percent Calculations

Calculations which require the answer to be expressed as a percent are common in the industry.

Examples include calculating the percent removal or reduction of solids and volatile solids, calculating chemical dosage requirements for disinfection, coagulation and flocculation and nutrient addition and a range of other applications.

The term "per cent" is derived from the Latin *per centum*, meaning "by the hundred".

A percent is simply a fraction where the denominator is always 100

$$\begin{aligned} \text{per cent, \%} &= \frac{\text{numerator}}{\text{denominator}} = \frac{\text{numerator}}{100} \\ \frac{25}{100} &= 25\% = 0.25 \end{aligned}$$

From the equation above we see that any value expressed as a percent can be expressed as a decimal value by moving the decimal point two (2) places to the left. Similarly, any decimal value can be expressed as a percent by moving the decimal place two (2) places to the right.

Two useful conversion factors to commit to memory are the fact that:

$$\mathbf{1\% = 10,000 \text{ mg/L and therefore, } 1 \text{ mg/L} = .0001\%}$$

Proofs:

If 1 litre of water weighs 1,000,000 mg.

$$\begin{aligned} 1\% \text{ of } \frac{1,000,000 \text{ mg}}{\text{L}} &= 0.01 \times \frac{1,000,000 \text{ mg}}{\text{L}} = 10,000 \text{ mg/L} \\ \frac{1 \text{ mg}}{\text{L}} &= \frac{1 \text{ mg}}{10^6 \text{ mg}} \times 100\% = 0.0001\% \end{aligned}$$

**What is the mass of solids contained in a sample of thickened waste activated sludge that contains 5% solids?**

Insert known value and solve:

$$5\% \text{ solids} = 5\% \times \frac{10,000 \text{ mg/L}}{1\%} = 50,000 \text{ mg/L}$$

or

$$5\% \times \frac{1 \text{ mg/L}}{0.0001\%} = 50,000 \text{ mg/L}$$

**What is 32% of 185?**

$$\frac{32}{100} \times 185 = 59.2 \quad \text{or} \quad 0.32 \times 185 = 59.2$$

**If 152 is 100% of a value what number represents 33.4% of that value?**

Let  $x$  = the unknown number (i.e.  $x$  = 33.4% of 152)

Solve for the unknown number

$$\frac{33.4}{100} \times 152 = 50.8 \quad \text{or} \quad 0.334 \times 152 = 50.8$$

Another way to solve this type of problem is to set up a ratio

$$\frac{152}{100\%} = \frac{x}{33.4\%}$$

Cross multiply and solve:

$$x = \frac{(152)(33.4\%)}{100\%} = 50.8$$

**If 1.27 kilograms of sodium hydroxide are mixed into 4.42 liters of water what is the percent sodium hydroxide in the mixture?**

$$\text{percent concentration} = \frac{\text{weight solute} \times 100\%}{\text{weight of solute} + \text{solvent}} \quad \text{or} \quad \frac{\text{weight of solute} \times 100\%}{\text{total weight of all products}}$$

Insert known values and solve:

$$\text{percent concentration} = \frac{1.27 \text{ kg} \times 100\%}{1.27 \text{ kg} + 4.42 \text{ kg}} = 22.3\%$$

**If 89 is 15% of something, what is 85%**

Write a ratio and solve for the unknown number:

$$\frac{89}{15\%} = \frac{\text{Unknown value}}{85\%}$$

Cross multiply and solve:

$$\text{Unknown value} = \frac{89 \times 85\%}{15\%} = 504$$

### Exponents and Powers of 10

In mathematics an exponent is the number to which the base number is to be multiplied by itself. In the example which follows the number 2 is the base and the exponent 3 indicates the number of times the base is to be multiplied by itself. Exponents are written as a superscript to the right of the number.

$$2^3 = (2)(2)(2) = 8$$

The expression  $b^2 = b \cdot b$  is called the square of  $b$ , The area of a square with side-length  $b$  is  $b^2$ .

The expression  $b^3 = b \cdot b \cdot b$  is called the cube of  $b$ , The volume of a cube with side-length  $b$  is  $b^3$ .

So  $3^2$  is pronounced "three squared", and  $2^3$  is "two cubed".

The exponent tells us how many copies of the base are multiplied together.

For example:  $3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$ .

The base 3 appears 5 times in the repeated multiplication, because the exponent is 5. Here, 3 is the base, 5 is the exponent, and 243 is the power or, more specifically, the fifth power of 3, or 3 raised to the fifth power, or 3 to the power of 5.

The word "raised" is usually omitted, and very often "power" as well, so  $3^5$  is typically pronounced "three to the fifth" or "three to the five".

### Powers of ten

In the base ten (decimal) number system, integer powers of 10 are written as the digit 1 followed or preceded by a number of zeroes determined by the sign and magnitude of the exponent. For example,  $10^3 = 1,000$  and  $10^{-4} = 0.0001$ .

Exponentiation with base 10 is used in scientific notation to denote large or small numbers. For instance, 299,792,458 m/s (the speed of light in vacuum, in metres per second) can be written as  $2.99792458 \times 10^8$  m/s and then approximated as  $2.998 \times 10^8$  m/s.

SI prefixes based on powers of 10 are also used to describe small or large quantities. For example, the prefix kilo means  $10^3 = 1,000$ , so a kilometre is 1,000 metres.

### Powers of 10 when the exponent is a positive number

$$8.64 \times 10^4$$

The small number 4 in the top right hand corner is the exponent.

$10^4$  is a shorter way of writing  $10 \times 10 \times 10 \times 10$ , or 10,000

$$8.64 \times 10^4 = 8.64 \times 10,000 = 86,400$$

10 to the power of any positive integer (i.e. 1, 2, 3, etc.) is a one followed by that many zeroes.

$$10^2 = 10 \times 10 = 100$$

$$10^3 = 10 \times 10 \times 10 = 1,000$$



## Metric Mathematics for Operators

$$10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1,000,000$$

### Powers of 10 when the exponent is a negative number:

$$8.64 \times 10^{-4}$$

The small number -4 in the top right hand corner is the exponent.

$$10^{-4} \text{ is a shorter way of writing } \frac{1}{10^4} = \frac{1}{10 \times 10 \times 10 \times 10} = \frac{1}{10,000} = 0.0001$$

Therefore:

$$8.64 \times 10^{-4} = \frac{8.64}{10 \times 10 \times 10 \times 10} = \frac{8.64}{10,000} = 0.000864$$

The decimal moves 4 places to the left

10 to the power of any negative integer (i.e. 1, 2, 3, etc.) is a one divided by the product of the power.

$$10^{-2} = \frac{1}{10 \times 10} = \frac{1}{100} = 0.01$$

$$10^{-3} = \frac{1}{10 \times 10 \times 10} = \frac{1}{1,000} = 0.001$$

$$10^{-4} = \frac{1}{10 \times 10 \times 10 \times 10} = \frac{1}{10,000} = 0.0001$$

### To recap -

If our exponent is a negative number, e.g.

$10^{-x}$  then the decimal place moves “x” places to the LEFT



For example

$$3.21 \times 10^{-3} \rightarrow \underline{.00} 3.21 \rightarrow 0.00321$$

←  
3 jumps to the left

$$4 \times 10^{-6} \rightarrow \underline{.00000} 4. \rightarrow 0.000004$$

←  
6 jumps to the left

If our exponent is a positive number, e.g.

$10^y$  then the decimal place moves “y” places to the RIGHT



For example

$$3.21 \times 10^3 \rightarrow 3.210 \rightarrow 3210$$

$\xrightarrow{\hspace{2cm}}$   
 3 jumps to the right

$$4 \times 10^6 \rightarrow 4000000. \rightarrow 4,000,000$$

$\xrightarrow{\hspace{2cm}}$   
 6 jumps to the right

### Multiplying and dividing by powers of ten

When we multiply two values expressed as powers of ten we add the exponents together

$$10^2 \times 10^3 = 10^{2+3} = 10^5$$

Example 1

$$125 \times 3,600 = 450,000$$

$$(1.25 \times 10^2) \times (3.6 \times 10^3) = 1.25 \times 3.6 \times 10^{2+3} = 4.5 \times 10^5 = 450,000$$

When we divide two values expressed as powers of ten we subtract the exponents

$$\frac{10^2}{10^3} = 10^{2-3} = 10^{-1}$$

Example 2

$$\frac{125}{3,600} = 0.034$$

$$\frac{1.25 \times 10^2}{3.6 \times 10^3} = \frac{1.25}{3.6} \times 10^{2-3} = 0.34 \times 10^{-1} = 0.034$$

*Note: normally, one would not use powers of ten notation for relatively small numbers such as those shown in the examples. The skill becomes useful in reducing some of the conversion factors used when converting from, say, milligrams per litre to kilograms per day*

### Geometry – Perimeter and Circumference, Area and Volume

Geometry is a branch of mathematics concerned with questions of shape, size, relative position of figures, and the properties of space.

Geometry arose independently in a number of early cultures as a practical way for dealing with lengths, areas, and volumes.

Some of the formulas that we still use today were first devised and recorded in the 3rd century BCE, by the Greek mathematician Euclid of Alexandria in his 13 volume treatise *Elements* which served as the main textbook for teaching mathematics (especially geometry) from the time of its publication until the late 19th or early 20th century.

Operators of wastewater treatment plants need to be familiar with the formulas for calculating areas, perimeters and volumes of a variety of geometric shapes. The shapes described below can be found in treatment process tanks and basins, in clarifiers, lagoons, trenches, storage hoppers and a variety of other locations and applications.



As the picture above shows, operators of wastewater treatment plants may be called upon to calculate the area of rectangles (aeration basins, primary clarifiers) and circles (clarifiers); the volume of polyhedrons (aeration basins, primary clarifiers, etc.), cylinders (clarifiers, digestors) and occasionally, a sphere (gas holder) or a storage hopper with a conical bottom and cylindrical barrel. Linear measurements such as the amount of perimeter fencing required or the circumference of circular process tank must also be calculated from time to time.

The tools to carry out these calculations are presented in the remainder of this section.

### Pi ( $\pi$ )

$\pi$  (sometimes written **pi**) is a mathematical constant which equals the ratio of a circle's circumference to its diameter.

$$\pi = \frac{\text{circumference}}{\text{diameter}} \approx 3.14$$

Many formulas in mathematics, science, and engineering involve  $\pi$ , which makes it one of the most important mathematical constants.

Pi is an irrational number, which means that its value cannot be expressed exactly as a fraction having integers in both the numerator and denominator (for example,  $22/7$ ). Consequently, its decimal representation never ends and never repeats. Reports on the latest, most-precise calculation of  $\pi$  are common. The record as of July 2016, stands at 13 trillion decimal digits.

The value used for  $\pi$  in all calculations in this book and on the EOCP exams is 3.14

### The constant 0.785

The number 0.785 often appears in formulas requiring the calculation of the area of a circle.

The equations:  $\text{Area} = 0.785(D)^2$  and  $\text{Area} = \pi r^2$  will give the same answer. Why?

Proof:

If  $\pi = 3.14$  and the radius of a circle is equal to one half the diameter i.e.  $r = D/2$

$$\text{Then Area} = \pi r^2 = \pi \left(\frac{D}{2}\right)^2 = \pi \frac{D^2}{4} = \frac{\pi D^2}{4} = \frac{3.14 D^2}{4} = 0.785 D^2$$

Because  $3.14 / 4 = 0.785$

Both formulas are correct but to avoid confusion operators should choose to use one or the other in all of their calculations. In this manual, the formula  $A = \pi r^2$  will be used throughout.

### Perimeter and Circumference

A **perimeter** is a path that surrounds a two-dimensional shape. The word comes from the Greek *peri* (around) and *meter* (measure). The term may be used either for the path or its length—it can be thought of as the length of the outline of a shape

In the wastewater industry this term is usually applied to shapes which are square or rectangular. A rectangle is any four sided shape having at least 1 right angle and a length which is longer than its width. A square is any four sided shape having at least 1 right angle and all four sides equal in length.

A practical application may be the calculation of the linear metres of fencing required to enclose a space.

The formula for calculating the perimeter of a rectangle is:

$$\text{Perimeter} = 2 \times (\text{length} + \text{width})$$

It is written as:

$$P = 2 \times (L + W) \text{ or } P = 2(L + W) \text{ or } P = 2L + 2W$$

**How many metres of fencing will be required to enclose a building lot that is 18 m wide by 45 m long?**

Known: Length = 45 m, Width = 18 m

Insert known values and solve:

$$P = 2 \times (L + W) = 2 \times (45 \text{ m} + 18 \text{ m}) = 2 \times (63 \text{ m}) = 126 \text{ metres}$$

The term **circumference** is used to refer to the distance around the outside of a circular or elliptical shape (its perimeter).

## Metric Mathematics for Operators

Calculation of the circumference of a circle requires the operator to know either its diameter (the distance across a circle at its widest point) or its radius (the distance from the center of a circle to its circumference or one half the diameter) and the value of the constant pi (3.14).

The formula for calculating the circumference of a circle is:

$$\text{circumference} = \pi \times \text{diameter} \text{ or } \pi \times 2 \times \text{radius}$$

It is written as:

$$C = \pi d \text{ or } C = 2\pi r$$

There is no simple formula with high accuracy for calculating the circumference of an ellipse. There are simple formulas but they are not exact, and there are exact formulas but they are not simple. Thankfully, there are not many elliptical aeration basins being constructed. The most accurate of the simple formulae for the circumference of an ellipse is:

$$\text{circumference} = \pi \times [3(a + b) - \sqrt{(3a + b)(a + 3b)}]$$

Where “a” and “b” are the major and minor axes of the ellipse and “a” is not more than three times the length of “b”. Even then, the formula is only accurate to  $\pm 5\%$ .

### **What is the circumference of a secondary clarifier with a diameter of 45 metres?**

Known: Diameter = 45 metres, pi ( $\pi$ ) = 3.14

Insert known values and solve:

$$C = \pi d = 3.14 \times 45 \text{ m} = 141.3 \text{ metres}$$

### **What is the circumference of a gravity thickener with a radius of 9 metres?**

Known: radius = 9 metres, pi ( $\pi$ ) = 3.14

Insert known values and solve:

$$C = 2\pi r = 2 \times 3.14 \times 9 \text{ m} = 56.52 \text{ metres}$$



## **Area**

The area of a geometrical shape such as a circle, square, rectangle or triangle is the space contained within the boundary of the shape (i.e. its perimeter). Two dimensions are required to calculate the area of a shape and that area is reported as “units” squared. In the metric system the units that are most commonly used are the square metre ( $\text{m}^2$ ) and the square centimetre ( $\text{cm}^2$ ). Large shapes such as land surveys and wastewater lagoons are often reported in units of hectares ( $10,000 \text{ m}^2$ ).

### Area of a Square or Rectangle

The area of a square or rectangle is equal to the product of one long side multiplied by one short side or in the case of a square by one side multiplied by another.

The formula for the area of a square or rectangle is:

$$\text{Area} = \text{Length} \times \text{Width}$$

It is written as:

$$A = L \times W \text{ or } A = LW \text{ or } A = (L)(W)$$

**What is the surface area of a clarifier that is 8 metres wide and 50 metres long?**

Known: Width = 8 metres, Length = 50 metres

Insert known values and solve;

$$\text{Area} = 8 \text{ m} \times 50 \text{ m} = 400 \text{ m}^2$$

### Area of a Triangle

The area of a triangle is equal to its base (any side of the triangle) multiplied by its height (perpendicular to, or at 90° to the base), divided by two (often written as multiplication by ½).

The formula is

$$\text{Area} = \frac{\text{Base} \times \text{Height}}{2}$$

It is written as:

$$A = \frac{(B) \times (H)}{2} \text{ or } \frac{1}{2}B \times H$$

**A compost pile is 7 metres wide and 3 metres high. What is its cross-sectional area?**

Known: Width (base) = 7 metres, Height = 3 metres

Insert known values and solve:

$$\text{Area} = \frac{7 \text{ m} \times 3 \text{ m}}{2} = 10.5 \text{ m}^2$$

### Area of a Trapezoid

Calculating the area of a trapezoid falls somewhere between calculating the area of a square and calculating the area of a triangle. Trapezoidal shapes found in the industry include trenches dug for the installation of pipelines and stock piles of materials such as wood chips, compost or soil.

The area of trapezoid is equal to the sum of its two sides divided by 2 times its height. The formula is:

$$\text{Area} = \frac{\text{side 1} + \text{side 2}}{2} \times \text{height}$$



**A pile of compost has a base 5 meters wide, a top 2.5 metres wide and a height of 2 meters. Calculate the cross-sectional area of the pile.**

Known: side 1 = 5 m, side 2 = 2.5 m, height = 2 m

Insert known values and solve

$$\text{Area} = \frac{\text{side 1} + \text{side 2}}{2} \times \text{height} = \frac{5 \text{ m} + 2.5 \text{ m}}{2} \times 2 \text{ m} = 7.5\text{m}^2$$

### Area of a Circle

The area of a circle is equal to its radius (the distance from the center to any outside point of the circle) squared, multiplied by the constant  $\pi$ .

The formula is:

$$\text{Area} = \pi \times (\text{radius})^2 \text{ or } \text{Area} = \pi \times \text{radius} \times \text{radius}$$

It is written as:

$$A = \pi r^2$$

**Calculate the surface area of a secondary clarifier which has a diameter of 45 metres.**

Known:  $\pi = 3.14$ , Radius =  $\frac{1}{2}$  of the diameter = 22.5 metres

Insert known values and solve:

$$\text{Area} = \pi(r)^2 = 3.14 \times (22.5 \text{ m})^2 = 3.14 \times 22.5 \text{ m} \times 22.5 \text{ m} = 1,589.6 \text{ m}^2$$

Circular shapes found in the industry include clarifiers, thickeners, wet wells, meter vaults and pipes.

### Area of a Cylinder

Calculating the area of a cylinder is a two-step operation. First the operator must calculate the circumference of the cylinder (i.e. the distance around the outside) and multiply that value by the height, depth or length of the cylinder as the case may be.

The practical application of this calculation is to determine the surface of area of a pipe, storage tank or reservoir in order to determine the quantity of paint or some other type of coating to be applied.

The equation is:

$$\text{Area} = \text{Circumference} \times \text{Height}$$

It is written as:

$$\text{Area} = C \times H \text{ or } \text{Area} = \pi \times D \times H$$

If the total area of a cylinder is to be calculated, as in calculating the surface area of a fuel tank then the two ends of the cylinder must also be accounted for and the formula becomes

$$\text{Total surface area} = \pi \times D \times H + 2 \times \pi r^2$$

**A 600 mm diameter force main that is 1.2 km long needs to be coated with an epoxy paint. Calculate the number of square meters of pipe that require coating.**

Step 1 – Convert to common units: 600 mm = 0.6 metres 1.2 km = 1,200 metres

Step 2 – Insert known values and solve

$$\text{Area} = \pi \times D \times H = 3.14 \times 0.6 \text{ m} \times 1,200 \text{ m} = 2,260.8 \text{ m}^2$$



### Area of a Sphere

This formula is provided in the EOCP handout with the notation that it might be used to calculate the surface area of an air bubble. It could also be used to calculate the surface area of a gas holder associated with an anaerobic digester.

The equation is:

$$\text{Area} = 4 \times \pi \times (\text{radius})^2$$

It is written:

$$\text{Area} = 4\pi r^2 \text{ or } \pi d^2 \text{ (where } d = \text{diameter)}$$

### Area of a Cone

This formula is provided in the ABC Canadian handout but not the EOCP handout. Its practical application would be to calculate the surface area of a conical section of a hopper or the floor of a clarifier, trickling filter or anaerobic digester in order to determine the amount of a coating needed.

The formula is:

$$\text{Area} = \pi \times \text{radius} \times \sqrt{(\text{radius})^2 + (\text{height})^2}$$

It is written:

$$\text{Area} = \pi \times r \times \sqrt{r^2 + h^2}$$

**A gravity thickener 10 m in diameter has a cone shaped floor. The cone is 1.5 m deep. A skim coat of concrete is to be applied to the floor. Calculate the number of square metres to be covered.**

Known: Radius =  $\frac{1}{2}$  of diameter = 5 metres, height = 1.5 metres

Insert known values and solve

$$\text{Area} = \pi \times r \times \sqrt{r^2 + h^2}$$

$$\text{Area} = 3.14 \times 5\text{m} \times \sqrt{(5\text{m})^2 + (1.5\text{m})^2}$$

$$\text{Area} = 3.14 \times 5\text{m} \times \sqrt{27.25 \text{ m}^2} = 82 \text{ m}^2$$



NOTE: *it is generally accepted that the math questions on a certification exam can be solved with a basic four function calculator, therefore, it is unlikely that any questions requiring the calculation of a square root will appear on the exam.*

### Area of an Irregular Shape

Occasionally it is necessary to calculate the area of an irregular shape such as a sewage lagoon. One way to do this is to break the shape into a number of shapes for which we have formulas (such as squares, rectangles or triangles). The area of each shape can be calculated, then added together to equal the area of the entire shape.

### Volume

A measure of the three dimensional space enclosed by a shape. As volume is a three-dimensional measurement, the units used to describe it need to have three dimensions as well. These units are reported as “units” cubed or cubic “units”. In the metric system volume is often expressed as cubic metres (m<sup>3</sup>), cubic centimetres (cm<sup>3</sup>) and liters (1,000 cm<sup>3</sup>). Large volumes are also reported as Megaliters (1 ML = 1,000,000 L = 1,000 m<sup>3</sup>).

In the water and wastewater industry operators often need to calculate the volume of a basin (rectangular), clarifier, digester or reservoir (cylinder), compost pile or stockpile (triangular) or a storage hopper (conical) or of a structure that is a combination of shapes (e.g. a digester with a cylindrical body and a conical floor)

### Volume of a Rectangular Tank

The volume of a box or cube is equal to its length, multiplied by its width, multiplied by its height, (depth or thickness). In the case of a cube, all three lengths are the same.

The formula is

$$\text{Volume} = \text{length} \times \text{width} \times \text{height}$$

It is written:

$$V = LWH \text{ or } V = (L)(W)(H) \text{ or } V = L \times W \times H$$

Sometimes the word “depth” and the letter “D” is substituted for height

**Calculate the volume of an aeration basin 50 metres long by 6 metres wide by 4.5 metres deep.**

Known: Length = 50 m, Width = 6 m, Depth = 4.5 m

Insert known values and solve:

$$V = LWD = 50 \text{ m} \times 6 \text{ m} \times 4.5 \text{ m} = 1,350 \text{ m}^3$$



### Volume of a Prism

The mathematical name for a three-dimensional shape that is triangular in cross-section is a prism. Examples of prismatic structures in the wastewater industry include spoil piles, compost piles and tanks which have a triangular cross section in their floors for the purposes of collecting sludge or grit.

The equation for the volume of a prism is one half its base times its height times its length

The formula is

$$\text{Volume of a prism} = \frac{\text{base} \times \text{height}}{2} \times \text{length}$$

It is written:

$$V = \frac{B \times H}{2} \times L$$

**Calculate the volume of a compost pile 3 metres high by 6 metres wide by 30 metres long.**

Known: Base = 6 metres, Height = 3metres, Length = 30 metres

Insert known values and solve:

$$V = \frac{B \times H}{2} \times L = \frac{6 \text{ m} \times 3 \text{ m}}{2} \times 30 \text{ m} = 270 \text{ m}^3$$

### Volume of a Cylinder

Calculation of the volume of a cylinder will probably be the most frequently used volume calculation after the calculation for the volume of a rectangular basin. Cylinders are found as circular clarifiers, reservoirs and water and sewer pipelines.

The volume of a cylinder is equal to the area of its circular base (the radius of the cylinder squared, multiplied by the constant  $\pi$ ), multiplied by the height

The formula is

$$\text{Volume} = \pi \times (\text{radius})^2 \times \text{height}$$

It is written:

$$V = \pi r^2 h \text{ or } V = \pi r^2 H$$



**What is the volume of a secondary clarifier that is 46 metres in diameter and 4.5 metres deep?**

Known: Diameter = 46 metres therefore radius =  $46 \div 2 = 23$  metres, depth = 4.5 metres

Insert known values and solve

$$V = \pi r^2 h = 3.14 \times (23 \text{ m})^2 \times 4.5 \text{ m} = 7,474.7 \text{ m}^3$$

### Volume of a Cone

Calculation of the volume of a cone is used less frequently but it may be required when calculating the volume of a storage hopper or the conical floor section of a digester, clarifier or trickling filter.

The volume of a cone is equal to the one third ( $\frac{1}{3}$ ) the area of its circular base (the radius of the cylinder squared, multiplied by the constant  $\pi$ ), multiplied by the height

The formula is:

$$\text{Volume} = \frac{\pi \times (\text{radius})^2 \times \text{height}}{3} \text{ or } V = \frac{\pi r^2 h}{3}$$

**Calculate the volume of conical hopper 2 metres deep and 1.5 metres in diameter.**

Known: diameter = 1.5 metres, therefore radius =  $1.5 \div 2 = 0.75$  m, depth = 2 metres

Insert known values and solve:

$$V = \frac{\pi r^2 h}{3} = \frac{3.14 \times (0.75 \text{ m})^2 \times 2 \text{ m}}{3} = 1.18 \text{ m}^3$$



**Volume of a lagoon (a frustrum)**

The correct name for a truncated pyramid is a frustrum. The EOCP handout provides a formula for calculating the volume of a lagoon which is a type of inverted truncated pyramid.

The volume of a frustrum is equal to one half (1/2) the average length times the average width times the depth.

The formula is:

$$\text{Volume} = \text{average length} \times \text{average width} \times \text{depth}$$

It is written:

$$V = \frac{L_{\text{top}} + L_{\text{bottom}}}{2} \times \frac{W_{\text{top}} + W_{\text{bottom}}}{2} \times \text{depth}$$

Where L = length and W = Width

**A lagoon measures 100 metres wide by 300 metres long on the surface, its bottom dimensions are 80 metres wide by 280 metres long. It is 2.5 metres deep. What is its volume?**

Known:  $L_t = 300$  m,  $L_b = 280$  m,  $W_t = 100$  m,  $W_b = 80$  m, Depth = 2.5 m

Insert known values and solve:

$$V = \frac{300 \text{ m} + 280 \text{ m}}{2} \times \frac{100 \text{ m} + 80 \text{ m}}{2} \times 2.5 \text{ m} = 65,250 \text{ m}^3$$

*Note: many other formulas exist for calculating the volume of a frustrum which give a more accurate result than the one used by the EOCP. For example:*

$$V = \frac{(A_{\text{top}} + A_{\text{bottom}} + \sqrt{A_{\text{top}} \times A_{\text{bottom}}})}{3} \times \text{depth}$$

*Yields a slightly different and more accurate answer.*

$$V = \frac{(30,000 \text{ m}^2 + 22,400 \text{ m}^2 + \sqrt{30,000 \text{ m}^2 \times 22,400 \text{ m}^2})}{3} \times 2.5 \text{ m} = 65,269 \text{ m}^3$$

## The Megaliter Shortcut

Many questions ask the operator to calculate the weight of a substance added to a process or wasted from a process per unit of time given the concentration of the substance in mg/L and the flow in either liters or cubic metres per unit of time.

Regardless of the substance, whether it be COD, BOD<sub>5</sub>, suspended solids, volatile solids, mixed liquor suspended solids or waste or return activated sludge the standard equation is:

$$\text{Loading, kg/day} = (\text{Flow, m}^3/\text{unit of time})(\text{Concentration, mg/L})$$

To solve the equation the operator inserts conversion factors and sets up the equation as follows:

$$\text{Loading} = \frac{X \text{ mg}}{\text{L}} \times \frac{Y \text{ m}^3}{\text{Time}} \times \frac{1 \text{ kg}}{10^6 \text{ mg}} \times \frac{10^3 \text{ L}}{\text{m}^3} = Z \text{ kg/time}$$

Where X = the concentration, Y = the flow and Z = the product after all of the math has been done

The benefit of the long form equation is that it allows the operator to “cancel out” words above and below the vinculum (the line which separates the numerator and denominator in a fraction) to see if the equation has even been set up properly before doing the math.

$$\text{Loading} = \frac{\cancel{X} \text{ mg}}{\cancel{\text{L}}} \times \frac{\cancel{Y} \text{ m}^3}{\text{Time}} \times \frac{1 \text{ kg}}{10^6 \cancel{\text{mg}}} \times \frac{10^3 \cancel{\text{L}}}{\cancel{\text{m}^3}} = Z \text{ kg/time}$$

As an alternative to setting up the equation long form, the operator can simply convert the flow to Megaliters (ML) [1 Megaliter = 1,000 cubic metres = 1,000,000 liters] and multiply by the concentration given in mg/L.

Why does this work? Consider that:

$$\frac{1 \text{ mg}}{\text{L}} = \frac{1,000 \text{ mg}}{1,000 \text{ L}} = \frac{1,000,000 \text{ mg}}{1,000,000 \text{ L}} = \frac{1 \text{ kg}}{\text{ML}}$$

Because 1,000,000 mg = 1 kg and 1,000,000 L = 1,000 m<sup>3</sup> = 1 ML

The following example illustrates the use of this shortcut.

**What is the loading on a basin if 2,500 cubic metres of a substance having a concentration of 180 mg/L is added per day?**

Example 1 - Insert known values and solve, long form

$$\text{Loading} = \frac{180 \text{ mg}}{\text{L}} \times \frac{2,500 \text{ m}^3}{\text{day}} \times \frac{1 \text{ kg}}{10^6 \text{ mg}} \times \frac{10^3 \text{ L}}{\text{m}^3} = 450 \text{ kg/day}$$

Example 2 – Megalitre shortcut

Step 1 – Convert 2,500 m<sup>3</sup> to Megalitres = 2,500 / 1,000 = 2.5 ML

Insert known values and solve

$$\text{Loading} = (180 \text{ mg/L})(2.5 \text{ ML/day}) = 450 \text{ kg/day}$$

**How many kilograms of solids are in an aeration basin 30 m long, 10 m wide and 3.5 m deep if the concentration of the MLSS is 2,450 mg/L?**

Step 1 – Calculate volume of aeration basin = LWD = (30)(10)(3.5) = 1,050 cubic metres = 1.05 ML

Insert known values and solve

$$\text{Loading} = \frac{2,450 \text{ mg}}{\text{L}} \times \frac{1,050 \text{ m}^3}{\text{day}} \times \frac{1 \text{ kg}}{10^6 \text{ mg}} \times \frac{10^3 \text{ L}}{\text{m}^3} = 2,572.5 \text{ kg}$$

or

$$\text{Solids} = (2,450 \text{ mg/L})(1.05 \text{ ML}) = 2,572.5 \text{ kg}$$

### Things That Are Equal to One

When setting up a problem it is often useful to insert conversion factors that will allow us to move from the units given in the problem to the units that are needed to answer the problem. Luckily in mathematics, multiplying and dividing by the number one (1) has no effect on the answer so the insertion of a conversion factor (so long as it is equal or equivalent to one) has no impact on the numerical answer but it will help us move from one unit to another. Some conversion factors that are equal to one include:

$\frac{1\text{m}^3}{1,000 \text{ L}}$	$\frac{10,000 \text{ m}^2}{\text{ha}}$	$\frac{1 \text{ kg}}{10^6 \text{ mg}}$	$\frac{1,000\text{g}}{\text{kg}}$
$\frac{1\text{ML}}{1,000 \text{ m}^3}$	$\frac{1,000 \text{ mg}}{\text{g}}$	$\frac{10^6 \text{ mg}}{1 \text{ Kg}}$	$\frac{1\text{kPa}}{1,000 \text{ Pa}}$

### Set up of Calculations in this Manual

Throughout this manual sample calculations will be written out each time and the units will be written with the problem. There are a number of benefits to be derived from this method but the main one is to identify units which may require conversion and to serve as a check on the final answer. When the units are written down it becomes easier to see which ones may be cancelled out. In each of the math examples the final answer has been rounded to either a whole number or one (1) decimal place.

## Before we get started

### 8 Simple Steps to Solving a Math Problem (and 1 more)

1. Make sure all the units are the same, you cannot multiply meters by millimetres or litres by kilograms. Look to the answer choices or the wording of the question to determine which units you should use.
2. If you think visually, make a sketch
3. Ensure you understand the question – read it then read it again.
4. Write down the things you know – what data did the question provide?  
Separate the wheat from the chaff, sometimes the question will provide you with more information than is really needed to solve it.
5. Find the applicable formula and write it down. The variables provided, information given and the units required in the answer will help you select the correct formula
6. Break the question into manageable sections, don't try and devise a super-formula.
7. Double check your calculations – calculator keys are small and fingers are big.
8. Does the answer look reasonable and are the units correct?

And 1 more

9. Time management – don't be afraid to flag the question and move onto the next one. Once you have finished all the other questions, then you can go back and work on the ones you flagged. Remember this, each question on a certification examination is worth exactly 1 mark. Don't burn up time that could have been spent answering questions to which you knew the answer. Pick the low-hanging fruit first and then go back for the hard ones. At the end of it all, if you haven't been able to achieve an answer than matches any of the ones given – guess. You have a 1 in 4 chance of being right and there is no penalty for being wrong.

Subject matter area	Class I Total # of questions	Class II Total # of questions	Math Based Questions
Laboratory analysis	10	15	Class I exam – 10 questions out of 100 total
Equipment evaluation and maintenance	25	20	
Equipment operation	25	25	
Treatment process monitoring, evaluation and adjustment	30	30	Class II exam – 12 questions out of 100 total
Security, safety and administrative procedures	10	10	
Totals	100	100	

## PART 1 – FORMULAS AND PRACTICE QUESTIONS – EOCP HANDOUT

The Association of Boards of Certification (ABC) developed a Canadian Formula / Conversion Table for wastewater treatment, industrial wastewater treatment, wastewater collection and laboratory exam in 2009. This 5 page handout, which is provided to exam candidates, lists formulas in alphabetical order. The Environmental Operators Certification Program (EOCP) also provides exam candidates with a 4 page handout which presents similar formulas arranged by topic.

The practice problems which follow are arranged in accordance with the layout of the EOCP handout first followed by practice problems for formulas given in the ABC handout but not the EOCP handout. Where the ABC and EOCP handouts contained similar formulas and headings they are incorporated in the EOCP sections.

Both handouts are included as Appendices 1 and 2.

### Solids (Mass) Calculation

Calculating the mass of solids added to or removed from a process is the most commonly performed calculation in wastewater mathematics. The task is embedded in almost every single math problem found on a certification examination.

The basic equation is:

$$\text{Mass} = \text{Concentration} \times \text{Flow} \quad \text{or} \quad \text{Mass} = \text{Concentration} \times \text{Volume}$$

In most applications of the formula some conversion factors will need to be applied to convert the values given to the values desired. E.g. from mg/L to kg or kg/m<sup>3</sup>

When the value desired is in kilograms the following formulas may be used:

$$\text{Mass, kg} = \frac{\text{Volume, m}^3 \times \text{Concentration, mg/L}}{1,000}$$

$$\text{Mass, kg} = \text{concentration, mg/L} \times \text{Flow, ML}$$

$$\text{Mass, kg} = \text{concentration, mg/L} \times \text{Volume, ML}$$

Where flow or volume are expressed in Megalitres, symbol ML (Megalitre = 10<sup>6</sup> L or 10<sup>3</sup> m<sup>3</sup>)

**Calculate the kilograms of MLSS under aeration in an aeration basin 50 m long by 5 m wide by 3 m deep if the MLSS concentration is 2,658 mg/L.**

Step 1 – Calculate the volume of the aeration basin

$$\text{Volume} = L \times W \times D = 50\text{m} \times 5\text{m} \times 3\text{m} = 750 \text{ m}^3$$

Insert known values and solve:

$$\begin{aligned} \text{Mass} &= \text{Concentration} \times \text{Flow} \\ \text{Mass} &= \frac{2,658 \text{ mg}}{\text{L}} \times 750\text{m}^3 \times \frac{1,000\text{L}}{\text{m}^3} \times \frac{1 \text{ kg}}{10^6\text{mg}} = 1,993.5 \text{ kg} \end{aligned}$$

In the equation above two conversion factors,  $\frac{1,000\text{L}}{\text{m}^3}$  and  $\frac{1\text{ kg}}{10^6\text{mg}}$  were required

Alternate formula:

$$\text{Mass, kg} = \text{concentration, mg/L} \times \text{Volume, ML}$$

Step 1 – Convert  $750\text{ m}^3$  to ML.  $750\text{ m}^3 = 0.75\text{ ML}$

Insert known values and solve

$$\text{Mass} = 2,658\text{ mg/L} \times 0.75\text{ML} = 1,993.5\text{ kg}$$

**Calculate the kilograms of BOD added to sequencing batch reactor each day if the influent BOD concentration is 168 mg/L and the flow is 75 L/s**

Known: Influent BOD = 168 mg/L, Flow = 75 L/s

$$\text{Mass} = \frac{168\text{ mg}}{\text{L}} \times \frac{75\text{ L}}{\text{second}} \times \frac{86,400\text{ seconds}}{\text{day}} \times \frac{1\text{ kg}}{10^6\text{mg}} = 1,088.6\text{ kg/day}$$

In the equation above two conversion factors,  $\frac{86,400\text{ seconds}}{\text{day}}$  and  $\frac{1\text{ kg}}{10^6\text{mg}}$  were required.

### Percent (%) Removal Calculation

After the solids equation, the percent removal calculation is one the most commonly calculated values.

Percent removal calculations whether for BOD<sub>5</sub>, TSS or VSS inform the operator of the efficiency of the unit process and provide information on the impact of the material removed on the next downstream process. (E.g. thickeners, digestors or dewatering equipment). The percent removal statement may sometimes be worded as percent reduction.

The equation is

$$\% \text{ removal efficiency} = \frac{(\text{parameter in} - \text{parameter out}) \times 100}{\text{parameter in}} \text{ or } \frac{(\text{in} - \text{out}) \times 100}{\text{in}}$$

**What is the removal efficiency of a primary clarifier if the influent TSS are 195 mg/L and the effluent TSS are 82 mg/L?**

Known: Influent TSS = 195 mg/L, effluent TSS = 82 mg/L

Insert known values and solve

$$\text{removal efficiency} = \frac{(\text{in} - \text{out}) \times 100}{\text{in}} = \frac{(195\text{ mg/L} - 82\text{ mg/L}) \times 100}{195\text{ mg/L}} = 57.9\%$$

### % Removal - Volatile Solids Destruction / Moisture Reduction

A modified version of the % removal formula is used when dealing with volatile solids reduction in a digester and the reduction in moisture content in digester sludge or a composting process. There have been a number of formulas used in the past to calculate volatile solids reduction in anaerobic digestors. Current practice is to use what is called the “Van Kleeck” formula for modern digestors.

In the formula all percent values are expressed as a decimal. E.g. 25% = 0.25

The formula is:



$$\% \text{ Volatile solids reduction} = \frac{(\text{Volatile solids in} - \text{Volatile solids out}) \times 100}{\text{Volatile solids in} - (\text{volatile solids in} \times \text{volatile solids out})}$$

It is often written as:

$$\% \text{VS reduction} = \frac{(\text{VS}_{\text{in}} - \text{VS}_{\text{out}}) \times 100}{\text{VS}_{\text{in}} - (\text{VS}_{\text{in}} \times \text{VS}_{\text{out}})} \quad \text{or} \quad \frac{(\text{in} - \text{out}) \times 100}{\text{in} - (\text{in} \times \text{out})}$$

**Calculate the % volatile solids reduction in an anaerobic digester which is fed primary sludge with a volatile solids content of 87% and produces a digested sludge with a volatile solids content of 59%**

Known: Volatile solids in = 87% = 0.87, Volatile solids out = 59% = 0.59

Step 1 – Insert known values and solve:

$$\% \text{VS reduction} = \frac{(\text{VS}_{\text{in}} - \text{VS}_{\text{out}}) \times 100}{\text{VS}_{\text{in}} - (\text{VS}_{\text{in}} \times \text{VS}_{\text{out}})}$$

$$\text{VS reduction} = \frac{(0.87 - 0.59) \times 100}{0.87 - (0.87 \times 0.59)} = \frac{28}{.87 - .51} = \frac{28}{.36} = 77.8\%$$

## Flow and Velocity Calculations

Operators need to know how to calculate flow and velocity in order to enhance settling in grit chambers or prevent settling in gravity sewers and force mains and to calculate appropriate chemical dosage.

Excessive velocities in pipe lines can accelerate wear.

Flow is measured as a volume (e.g. litre, cubic metre, megalitre) per unit of time (e.g. second, minute, hour, day).

A number of formulas are used:

$$\text{Flow (Q) in an open channel} = \text{Area, (width} \times \text{depth)} \times \text{Velocity}$$

$$\text{Flow (Q) in a pipe} = \text{Area, } (\pi r^2) \times \text{Velocity}$$

Both formulas can be rearranged to solve for velocity

$$\text{Velocity} = \frac{\text{Flow (Q)}}{\text{Area}}$$

*Note: this formula applies only to incompressible fluids like water and wastewater. A different formula would be used to calculate the velocity of say, an air stream.*

**What is the velocity in metres per second of water flowing through a channel that is 2.5 metres wide by 1 metre deep if the rate of flow is 0.9 cubic metres per second?**

Known: Channel width = 2.5 m, Channel depth = 1m, Flow = 0.9 m<sup>3</sup>/s

The equation for flow in a channel (or pipe) is Flow = Area / Velocity

Insert known values and solve:

$$\text{Velocity} = \frac{\text{Flow (Q)}}{\text{Area}} = \frac{0.9 \text{ m}^3/\text{s}}{2.5 \text{ m} \times 1 \text{ m}} = 0.36 \text{ m/s}$$

**What is the flow in liters per second in a pipe with a diameter of 20 centimetres if the water is flowing at a velocity of 0.5 metres per second? (assume that the pipe is flowing full)**

Known: Velocity = 0.5 m/s, pipe diameter = 20 cm = 0.2 m, radius = 1/2 diameter = 0.1 m

Step 1 – Calculate the area of the pipe in square metres.

$$\text{Area} = \pi \times (\text{radius})^2 = 3.14 \times 0.1 \text{ m} \times 0.1 \text{ m} = 0.0314 \text{ m}^2$$

Step 2 – Insert known values and solve

$$\text{Flow} = \text{Area} \times \text{Velocity} = 0.0314 \text{ m}^2 \times 0.5 \text{ m/s} = 0.016 \text{ m}^3/\text{s}$$

Step 3 – Convert flow to L/s

$$\frac{0.016 \text{ m}^3}{\text{s}} \times \frac{1,000 \text{ L}}{\text{m}^3} = 16 \text{ L/s}$$

**Determine the required length of a manually cleaned grit channel if the depth of the channel is 0.4 m and the flow velocity is 0.29 m/s. Jar tests for this plant determined that grit settles at the rate of 23 millimetres per second.**

There are two ways to solve this question.

Method 1

The equation is:

$$\text{Required length, m} = \frac{\text{depth, m} \times \text{velocity, m/s}}{\text{settling rate, m/s}}$$

Step 1 – Convert all units to the same unit of measure

$$\text{Settling rate} = \frac{23 \text{ mm}}{\text{s}} \times \frac{1 \text{ m}}{1,000 \text{ mm}} = 0.023 \text{ m/s}$$

Insert known values and solve

$$\text{Channel length} = \frac{0.4 \text{ m} \times 0.29 \text{ m/s}}{0.023 \text{ m/s}} = 5 \text{ m}$$

Method 2

Step 1 – Calculate the time required for a particle to sink 0.4 metres

$$\text{Time} = \frac{\text{Distance, m}}{\text{Velocity, m/s}} = \frac{0.4 \text{ m}}{0.023 \text{ m/s}} = 17.4 \text{ seconds}$$

Step 2 – Calculate the distance travelled in the required time

$$\text{Distance (channel length)} = \text{Velocity} \times \text{Time}$$

$$\text{Channel length} = \frac{0.29 \text{ m}}{\text{s}} \times 17.4 \text{ s} = 5 \text{ m}$$

### Detention Time (or Hydraulic Retention Time) Calculations

Detention time measures the length of time a particle of water remains in a tank, basin, pond or pipe. i.e. the time elapsed from the moment a particle enters the tank to the moment when it leaves the tank. It is often measured for lagoons, aeration basins, clarifiers, wet wells, UV or chlorine contact chambers, force mains and outfalls.

The equation for detention time is:

$$\text{Detention time} = \frac{\text{Volume}}{\text{Flow}}$$

**What is the detention time in days for an aerated lagoon that is 120 metres long, 50 metres wide and 1.45 metres deep if it receives a flow of 223 cubic metres per day?**

Known: Length = 120 m, width = 50 m, depth = 1.45 m, Flow = 223 m<sup>3</sup>/day

Step 1 – Calculate the volume of the lagoon

$$\text{Volume} = L \times W \times D = 120 \text{ m} \times 50 \text{ m} \times 1.45 \text{ m} = 8,700 \text{ m}^3$$

Insert known values and solve

$$\text{Detention time} = \frac{\text{Volume}}{\text{Flow}} = \frac{8,700 \text{ m}^3}{223 \text{ m}^3/\text{day}} = 39 \text{ days}$$

**A treatment plant has a clarifier 30 metres in diameter by 3.65 metres deep. What is the detention time in the clarifier in hours if the flow is 180 liters per second?**

This problem requires that the volume of the clarifier and the flow in cubic metres per hour be calculated before the detention time can be calculated.

Known: Clarifier diameter = 30 m, radius = 30 ÷ 2 = 15 m, depth = 3.65 m, Flow = 180 L/s

Step 1 – Calculate the volume of the clarifier and the flow in cubic metres per hour

Insert known values and solve

$$\text{Volume} = \pi r^2 h = 3.14 \times 15 \text{ m} \times 15 \text{ m} \times 3.65 \text{ m} = 2,579 \text{ m}^3$$

$$\text{Flow} = \frac{180 \text{ L}}{\text{s}} \times \frac{3,600 \text{ s}}{\text{hr}} \times \frac{1 \text{ m}^3}{1,000 \text{ L}} = 648 \text{ m}^3/\text{hour}$$

Step 2 – Insert known values and calculate the detention time

$$\text{Detention time} = \frac{\text{Volume}}{\text{Rate of Flow}} = \frac{2,579 \text{ m}^3}{648 \text{ m}^3/\text{hr}} = 4 \text{ hours}$$

**A 200 mm diameter sludge line needs to be flushed. If the line is 75 m long how many minutes will it take to flush the line at a flow of 3.5 L/second?**

Known: Diameter = 200 mm, radius =  $200 \text{ mm} \div 2 = 100 \text{ mm} = 0.1 \text{ m}$ , Length = 75 m, Flow = 3.5 L/s

Step 1 – Calculate the volume of the pipeline

$$\text{Volume} = \pi r^2 l = 3.14 \times 0.1 \text{ m} \times 0.1 \text{ m} \times 75 \text{ m} = 2.355 \text{ m}^3 = 2,355 \text{ L}$$

Step 2 – Insert known values and solve

$$\text{Flushing time} = \frac{\text{Volume}}{\text{Rate of Flow}} = \frac{2,355 \text{ L}}{3.5 \text{ L/s}} = 673 \text{ s} = 11.2 \text{ minutes}$$

Alternate equation

$$\text{Flushing time} = 2.355 \text{ m}^3 \times \frac{1,000 \text{ L}}{\text{m}^3} \times \frac{1 \text{ s}}{3.5 \text{ L}} \times \frac{1 \text{ minute}}{60 \text{ s}} = 11.2 \text{ minutes}$$

### Hydraulic Loading Rate Calculations

Hydraulic loading rates are important control parameters for clarifiers, rotating biological contactors, trickling filters and activated sludge processes. They can be used to determine sludge withdrawal rates and contact times between food and microorganisms.

Two different equations are used depending on the process being monitored. They are:

$$\text{Hydraulic loading rate} = \frac{\text{Flow}}{\text{Volume}} \quad \text{or} \quad \text{Hydraulic loading rate} = \frac{\text{Flow}}{\text{Surface area}}$$

Values obtained from these equations are often expressed as:

$$\text{m}^3/\text{m}^2/\text{day} \quad \text{or} \quad \frac{\text{m}^3}{\text{m}^2/\text{day}}$$

### Aeration basins, Rotating biological contactors, Filter flow and filter backwash

**Calculate the hydraulic loading rate on a circular clarifier 27.5 metres in diameter if it receives a flow of 10,900 m<sup>3</sup>/day at a MLSS concentration of 2,825 mg/L.**

Known: Diameter = 27.5m, radius =  $27.5 \div 2 = 13.75\text{m}$ , Flow = 10,900 m<sup>3</sup>/day, MLSS = 2,825 mg/L

Clarifier hydraulic loading rates are expressed as cubic metres per day per square metre.

Step 1 – Calculate the surface area of the clarifier

$$\text{Area} = \pi r^2 = 3.14 \times 13.75\text{m} \times 13.75\text{m} = 593.7\text{m}^2$$

Insert known values and solve

$$\text{Hydraulic loading rate} = \frac{\text{Flow}}{\text{Surface area}} = \frac{10,900 \text{ m}^3/\text{day}}{593.7 \text{ m}^2} = 18.4 \text{ m}^3/\text{m}^2/\text{day}$$

**What is the hydraulic loading rate in m<sup>3</sup>/m<sup>2</sup>/day on a trickling filter 25 metres in diameter if the influent flow is 15,000 m<sup>3</sup>/day and the recirculation rate is 1,500 m<sup>3</sup>/day?**

Known: Diameter = 25 m, radius = 25 ÷ 2 = 12.5 m, Flow = 15,000 m<sup>3</sup>/day + 1,500 m<sup>3</sup>/day = 16,500 m<sup>3</sup>/day

Step 1 – Calculate surface area of trickling filter

$$\text{Surface area} = \pi r^2 = 3.14 \times 12.5 \text{ m} \times 12.5 \text{ m} = 490.6 \text{ m}^2$$

Insert known values and solve

$$\text{Hydraulic loading rate} = \frac{\text{Flow}}{\text{Surface area}} = \frac{16,500 \text{ m}^3/\text{day}}{490.6 \text{ m}^2} = 33.6 \text{ m}^3/\text{m}^2/\text{day}$$

**What is the hydraulic loading rate in m<sup>3</sup>/m<sup>3</sup>/day on an aeration basin 50 m long, 10 m wide and 3.5 m deep if primary effluent is flowing into the basin at a rate of 416 m<sup>3</sup>/hour?**

Step 1 – Calculate the volume of the basin

$$\text{Volume} = L \times W \times D = 50 \text{ m} \times 10 \text{ m} \times 3.5 \text{ m} = 1,750 \text{ m}^3$$

Step 2 – Calculate the daily flow

$$\text{Flow} = \frac{416 \text{ m}^3}{\text{hr}} \times \frac{24 \text{ hr}}{\text{day}} = 9,984 \text{ m}^3/\text{day}$$

Insert calculated values and solve:

$$\text{Hydraulic loading rate} = \frac{\text{Flow}}{\text{Volume}} = \frac{9,984 \text{ m}^3/\text{day}}{1,750 \text{ m}^3} = 5.7 \text{ m}^3/\text{m}^3/\text{day}$$

**Calculate the hydraulic loading rate on a rotating biological contactor. The influent flow is 1000 m<sup>3</sup>/day with a BOD of 185 mg/L. The RBC has 400 disks each 3.5 metres in diameter mounted on its shaft.**

The equation is:

$$\text{Hydraulic loading rate} = \frac{\text{Flow}}{\text{Surface area}}$$

Step 1 – Calculate surface area of disks

$$\text{Area} = \pi r^2 \times 2 \times \text{number of disks} = 3.14 \times 2 \times (1.75 \text{ m})^2 \times 400 = 7,693 \text{ m}^2$$

Step 2 – Insert calculated values and solve

$$\text{Hydraulic loading rate} = \frac{\text{Flow}}{\text{Surface area}} = \frac{1,000 \text{ m}^3/\text{day}}{7,693 \text{ m}^2} = 0.13 \text{ m}^3/\text{m}^2/\text{day}$$

### Litres per Capita per day

Designers of wastewater treatment plants will select a litres per capita per day flow as a data point in the design of a plant. Operators can compare the population served to the flow to determine whether infiltration and inflow is increasing or decreasing over time.

The formula is:

$$\text{Litres per capita per day, Lcd} = \frac{\text{Volume of wastewater treated, L/day}}{\text{Population served.}}$$

A small package treatment plant receives a flow of 1,200 m<sup>3</sup>/day from a population of 3,750. What is the per capita per day flow?

$$\text{Litres per capita per day, Lcd} = \frac{1,200 \text{ m}^3 \times 1,000 \text{ L/m}^3}{3,750} = 320$$

### Hydraulic Overflow Rate

#### Surface Overflow Rate (SOR) Calculations

The surface overflow rate (sometimes called the rise rate) is another measure used to determine the loading on a clarifier. As the SOR increases the velocity with which the water moves up and out of the clarifier increases and particles which may settle slowly can be carried over the weir leading to a decrease in effluent quality and percent removal rates. Surface overflow rates are typically expressed in units of volume/time/area (e.g. litres/second/square metre or cubic metres/day/square metre).

The equation is:

$$\text{Surface overflow rate (SOR)} = \frac{\text{Flow}}{\text{Surface area}}$$

It is written:

$$\text{SOR} = \frac{Q}{A}$$

**What is the surface overflow rate in cubic metres/day/square metre in a basin that is 37 metres long and 11 metres wide if the flow is 4,985 cubic metres per day?**

Known: Length = 37 m, width = 11 m, Flow = 4,985 m<sup>3</sup>/day

Step 1 – Calculate area of basin:

$$\text{Area} = L \times W = 37 \text{ m} \times 11 \text{ m} = 407 \text{ m}^2$$

Insert known values and solve

$$\text{Surface overflow rate (SOR)} = \frac{\text{Flow}}{\text{Surface area}} = \frac{4,985 \text{ m}^3/\text{day}}{407 \text{ m}^2} = 12.2 \text{ m}^3/\text{m}^2/\text{day}$$

**Calculate the surface overflow rate in cubic metres/day/square metre for a clarifier 22.5 metres in diameter if the flow is 5,910 cubic metres per day.**

Known: Diameter = 22.5 m, radius =  $22.5 \text{ m} \div 2 = 11.25$ , Flow = 5910 m<sup>3</sup>/day

Step 1 – Calculate the surface area of the clarifier

$$\text{Area} = \pi r^2 = 3.14 \times 11.25 \text{ m} \times 11.25 \text{ m} = 397.4 \text{ m}^2$$

Step 2 - Insert known values and solve

$$\text{Surface overflow rate (SOR)} = \frac{\text{Flow}}{\text{Surface area}} = \frac{5,910 \text{ m}^3/\text{day}}{397.4 \text{ m}^2} = 14.9 \text{ m}^3/\text{m}^2/\text{day}$$

### **Weir Overflow Rate (WOR) Calculations**

The weir overflow rate is one of the measures used to determine the loading on a clarifier. As overflow rates increase the velocity with which the water moves over the weir increases and particles which may settle slowly can be carried over the weir leading to a decrease in effluent quality and percent removal rates. Weir overflow rates are typically expressed in units of volume/time/length (e.g. litres/second/metre).

The formula for the weir overflow rate (WOR) is:

$$\text{Weir overflow rate} = \frac{\text{Flow}}{\text{Weir length}}$$

Values obtained from these equations are often expressed as:

$$\text{L/m/second or } \frac{\text{L}}{\text{m/second}}$$

**A rectangular clarifier has a total weir length of 60 metres. What is the WOR in liters/second/metre if the daily flow is 4,200 m<sup>3</sup>/day?**

Known: Weir length = 60 m, Flow = 4,200 m<sup>3</sup>/day

Step 1 – convert flow from cubic metres per day to liters per second

$$\text{Flow} = \frac{4,200 \text{ m}^3}{\text{day}} \times \frac{1,000 \text{ L}}{\text{m}^3} \times \frac{1 \text{ day}}{86,400 \text{ s}} = 48.6 \text{ L/s}$$

Insert known values and solve

$$\text{Weir overflow rate} = \frac{\text{Flow}}{\text{Weir length}} = \frac{48.6 \text{ L/s}}{60 \text{ m}} = 0.8 \text{ L/m/s}$$

**A circular clarifier has a diameter at the weir of 32 metres. If the daily flow is 7,600 cubic metres per day what is the WOR in cubic metres/day/metre of weir length?**

Known: Diameter = 32 m, Flow = 7,600 m<sup>3</sup>/day

Step 1 – Calculate the weir length

$$\text{Circumference} = \pi d = 3.14 \times 32 \text{ m} = 100.5 \text{ m}$$

Insert known values and solve

$$\text{Weir overflow rate} = \frac{\text{Flow}}{\text{Weir length}} = \frac{7,600 \text{ m}^3/\text{day}}{100.5 \text{ m}} = 75.6 \text{ m}^3/\text{m}/\text{day}$$

## Chemical Feed Rate / Chlorine Feed Rate

### Chlorine Dosage

**Service water is being disinfected with a 5.0% sodium hypochlorite solution (NaOCl). The hypochlorinator is pumping at a rate of 145 litres per day. What is the sodium hypochlorite dosage if the service water feed pump is delivering at a rate of 22 litres per second?**

Known: NaOCl feed: 145 L/s at 5% strength (50,000 mg/L, 0.05 decimal), Feed water: 22 L/s

The formula is:

$$\text{Chlorine dosage} = \frac{\text{decimal concentration of chlorine solution} \times \text{volume, L/day} \times 1000}{\text{volume of water, m}^3/\text{day}}$$

Step 1 – Calculate the amount of service water produced in a day in cubic metres

$$\text{Volume} = \frac{22 \text{ L}}{\text{s}} \times \frac{86,400 \text{ s}}{\text{d}} \times \frac{1 \text{ m}^3}{1,000 \text{ L}} = 1,900.8 \text{ m}^3/\text{day}$$

Step 2 – Insert known values and solve

$$\text{Chlorine dosage} = \frac{0.05 \times 145 \text{ L/day} \times 1000}{1,900.8 \text{ m}^3/\text{day}} = 3.8 \text{ mg/L}$$

Alternate method:

Step 1 – Calculate the amount of sodium hypochlorite fed in a day

$$\text{NaOCl fed} = \frac{145 \text{ L}}{\text{day}} \times \frac{5\%}{100\%} \times \frac{1 \text{ kg}}{\text{L}} \times \frac{10^6 \text{ mg}}{\text{kg}} = 7,250,000 \text{ mg/day}$$

or

$$\text{NaOCl fed} = \frac{145 \text{ L}}{\text{day}} \times \frac{50,000 \text{ mg}}{\text{L}} = 7,250,000 \text{ mg/day}$$

Step 2 – Calculate the daily flow

$$\text{Daily flow} = \frac{22 \text{ L}}{\text{s}} \times \frac{86,400 \text{ s}}{\text{day}} = 1,900,800 \text{ L/day}$$



Step 3 – Calculate dose

$$\text{Dose} = \frac{\text{Chemical fed}}{\text{flow}} = \frac{7,250,000 \text{ mg/day}}{1,900,800 \text{ L/day}} = 3.8 \text{ mg/L}$$

**What is the chlorine concentration if 68 kilograms of chlorine is added to a flow of 6,000 cubic metres?**

The formula is:

$$\text{Chlorine concentration} = \frac{\text{weight, kg} \times 1000}{\text{volume of water, m}^3}$$

Insert known values and solve:

$$\text{Chlorine concentration} = \frac{68 \text{ kg} \times 1000}{6,000 \text{ m}^3} = 11.33 \text{ mg/L}$$

### **Chlorine dose, chlorine demand, chlorine residual calculations**

Calculation of chlorine dose, demand or residual can be accomplished using the following basic set of equations:

$$\text{Chlorine dose} = \text{chlorine demand} + \text{chlorine residual}$$

$$\text{Chlorine demand} = \text{chlorine dose} - \text{chlorine residual}$$

$$\text{Chlorine residual} = \text{chlorine dose} - \text{chlorine demand}$$

**If the chlorine dose is 12.2 mg/L and the chlorine residual is 0.5 mg/L, what is the chlorine demand?**

$$\text{Chlorine demand} = \text{chlorine dose} - \text{chlorine residual} = 12.2 \text{ mg/L} - 0.5 \text{ mg/L} = 11.7 \text{ mg/L}$$

**If the chlorine dose is 8.05 mg/L and the chlorine demand is 7.43 mg/L, what is the chlorine residual?**

$$\text{Chlorine residual} = \text{chlorine dose} - \text{chlorine demand} = 8.05 \text{ mg/L} - 7.43 \text{ mg/L} = 0.62 \text{ mg/L}$$

**A treatment plant is using 200 kg/d of chlorine gas. If the chlorine demand is 2.5 mg/L and the chlorine residual is 0.2 mg/L what is the daily flow in cubic metres?**

Known: Chlorine demand = 2.5 mg/L, chlorine residual = 0.2 mg/L, chlorine consumed = 200 kg/day

Step 1 – find the chlorine dosage:

$$\text{Chlorine dose} = \text{chlorine demand} + \text{chlorine residual} = 2.5 \text{ mg/L} + 0.2 \text{ mg/L} = 2.7 \text{ mg/L}$$

Step 2 – calculate the flow by rearranging the weight formula

$$\text{If weight used} = \text{concentration} \times \text{flow} \text{ then } \text{flow} = \frac{\text{weight used}}{\text{concentration}}$$

Insert known values and solve

$$\text{flow} = \frac{\text{weight used}}{\text{concentration}} = \frac{200 \text{ kg}}{\text{day}} \times \frac{1 \text{ L}}{2.7 \text{ mg}} \times \frac{1 \text{ m}^3}{1,000 \text{ L}} \times \frac{10^6 \text{ mg}}{\text{kg}} = 74,074 \text{ m}^3/\text{day}$$

### Chemical Feed Rate Calculations

Accurate knowledge of the amounts of a chemical required for process control will prevent process upsets and ensure that the desired effect is obtained. Records of dosage and usage can be used for budgeting and cost control as well

Two different formulas are provided in the handouts for the calculation of chemical feed rates. They are:

$$\text{Chemical feed rate, L/day} = \frac{\text{dosing concentration, mg/L} \times \text{Flow}}{\text{chemical concentration as a decimal} \times \text{density} \times 1,000}$$

$$\text{Chemical feed rate, mL/minute} = \frac{\text{flow, m}^3/\text{day} \times \text{dose, mg/L}}{\text{chemical density, g/cm}^3 \times \text{chemical concentration as a decimal} \times 1,440}$$

**How many liters per day of a 12.0% sodium hypochlorite (NaOCl) solution are needed to disinfect a flow of 2,500 cubic metres if the dosage required is 8.5 mg/L? A 12% solution of NaOCl weighs 1.19 kg/L**

$$\text{Chemical feed rate, L/day} = \frac{\text{dosing concentration, mg/L} \times \text{Flow}}{\text{chemical concentration as a decimal} \times \text{density} \times 1,000}$$

$$\text{Chemical feed rate, L/day} = \frac{8.5 \text{ mg/L} \times 2,500 \text{ m}^3/\text{day}}{0.12 \times 1.19 \times 1,000} = 148.8 \text{ L/day}$$

Alternate method:

Step 1 – Calculate the kilograms of sodium hypochlorite required per day

$$\text{NaOCl required} = \frac{8.5 \text{ mg}}{\text{L}} \times \frac{2,500 \text{ m}^3}{\text{day}} \times \frac{1 \text{ kg}}{10^6 \text{ mg}} \times \frac{10^3 \text{ L}}{\text{m}^3} = 21.25 \text{ kg/day}$$

But the solution only contains 12% NaOCl

Step 2 – Calculate how many litres of solution are required

$$\text{NaOCl required} = \frac{21.25 \text{ kg}}{\text{day}} \times \frac{100 \text{ kg solution}}{12 \text{ kg NaOCl}} \times \frac{1 \text{ L}}{1.19 \text{ kg}} = 148.8 \text{ L/day}$$

**If a chemical feed pump is delivering 46.56 liters per day what is the feed rate in milliliters per minute?**

Known: Chemical used = 46.56 L/day, 1 day = 1,440 minutes, 1 litre = 1,000 mL

The equation is:

$$\text{Feed rate} = \frac{\text{Volume}}{\text{Time}}$$

The equation is: feed rate = volume / time

Known: 1 day = 1,440 minutes 1 Liter = 1,000 mL 1 kilogram = 1,000 grams

Insert known values and solve

$$\text{Feed rate} = \frac{\text{Volume}}{\text{Time}} = \frac{46.56 \text{ L} \times 1,000 \text{ mL/L}}{1,440 \text{ minutes/day}} = 32.3 \text{ mL/minute}$$

**A BNR facility uses an alum solution that contains 655 grams of alum per liter. If the target dose is 11.0 mg/L of alum, what should the feed pump be set at in liters per day if the flow is 4,500 cubic metres per day?**

Known: Flow = 4,500 m<sup>3</sup>/day = 4.5 ML, Dose = 11 mg/L, Alum concentration = 655 g/L

Step1 – Calculate how many kilograms of alum are required each day

The equation is:

$$\text{kg Alum required} = \text{Flow} \times \text{Concentration}$$

Insert known values and solve:

$$\text{kg Alum required} = \frac{11.0 \text{ mg}}{\text{L}} \times \frac{4,500 \text{ m}^3}{\text{day}} \times \frac{1 \text{ kg}}{10^6 \text{ mg}} \times \frac{10^3 \text{ L}}{\text{m}^3} = 49.5 \text{ kg/day}$$

Step 2 – Calculate the amount of alum solution required

$$\text{If } \text{kg Alum required} = \text{Flow} \times \text{Concentration} \text{ then } \text{Flow} = \frac{\text{kg Alum required}}{\text{Concentration}}$$

$$\text{Alum solution required} = \frac{49.5 \text{ kg}}{\text{day}} \times \frac{1 \text{ L}}{0.655 \text{ kg Alum}} = 75.6 \text{ L/day}$$

**How much sulfur dioxide (SO<sub>2</sub>) in kg/day needs to be applied to dechlorinate an effluent if the flow is 6,700 cubic metres per day, the chlorine residual is 0.2 mg/L and the sulfur dioxide dose must be 2.0 mg/L higher than the chlorine residual?**

Step 1 – Calculate the required SO<sub>2</sub> dose

$$\text{SO}_2 \text{ dose} = \text{safety factor} + \text{chlorine residual} = 2.0 \text{ mg/L} + 0.2 \text{ mg/L} = 2.2 \text{ mg/L SO}_2$$

Step 2 – Calculate the kilograms per day of SO<sub>2</sub> required

$$\text{SO}_2 \text{ dose} = \text{flow} \times \text{concentration} = 6.7 \text{ ML} \times 2.2 \text{ mg/L} = 14.7 \text{ kg/day}$$

*Note: Stoichiometrically only 1 mg/L of SO<sub>2</sub> is required to neutralize 1 mg/L of chlorine residual, use of excess SO<sub>2</sub> wastes money and can depress the pH of the effluent*

### Ratio Calculations

**A metering pump discharges 30 mL of a chemical at a speed setting of 22% and a stroke setting of 10%. If the amount of chemical needed increases from 30 mL to 45 mL what should the speed setting be changed to?**

This problem can be solved using a ratio, as follows:

Known: Initial speed setting = 22%, Initial dosage = 30 mL, Required dosage = 45 mL

Unknown: New speed setting

Set up the problem using the names of the variables.

$$\frac{\text{Initial speed setting, Percent}}{\text{Initial Chemical dosage, mL}} = \frac{\text{New speed setting, Percent}}{\text{Required dosage, mL}}$$

Rearrange equation, insert known values and solve

$$\frac{22\%}{30 \text{ mL}} = \frac{\text{New speed setting, Percent}}{45 \text{ mL}}$$

$$\text{New speed setting} = \frac{(22\%)(45 \text{ mL})}{30 \text{ mL}} = 33\%$$

**A chemical dosage of 5.5 mg/L is required to treat a flow of 2.9 ML/day. If the flow increases to 4.2 ML/day what should the dosage be increased to assuming all other parameters remain the same?**

Known: Initial flow = 2.9 ML/day, New flow = 4.2 ML/day, Initial chemical dosage 5.5 mg/L

Unknown = new dosage required

Set up the ratio

$$\text{If } \frac{\text{Initial flow}}{\text{Initial dosage}} = \frac{\text{New flow}}{\text{New dosage}} \text{ then New dosage} = \frac{\text{new flow} \times \text{initial dosage}}{\text{initial flow}}$$

Insert known values and solve

$$\text{New dosage} = \frac{\text{new flow} \times \text{initial dosage}}{\text{initial flow}} = \frac{4.2 \text{ ML/d} \times 5.5 \text{ mg/L}}{2.9 \text{ ML/d}} = 7.9 \text{ mg/L}$$

### Organic Loading Rate

Organic loading rates provide the operator with an indication of the amount of food entering a biological process. The concept is more generally applied to wastewater lagoons (kg/ha/day), trickling filters (kg/m<sup>3</sup>/day) and rotating biological contactors (g/m<sup>2</sup>/day) than it is to the activated sludge process which uses the concept of food to microorganism ratio.

The general equation for organic loading is:

$$\text{Organic loading rate} = \frac{\text{Flow} \times \text{concentration}}{\text{area}} \text{ or } \frac{\text{Flow} \times \text{concentration}}{\text{volume}}$$

For most applications the mass being added is either total suspended solids (TSS) or biochemical oxygen demand (BOD)

Mass loadings for clarifiers, lagoons, and rotating biological contactors are area based while mass loadings for aeration basins, trickling filters and digestors are volume based.

### Aeration Basins, Rotating Biological Contactors, Trickling Filters, Clarifiers

**Calculate the number of kilograms of TSS per square metre added per day to a primary clarifier. The clarifier is 50 m long, 5 m wide and 3.5 m deep. The flow is 14,500 m<sup>3</sup>/day and the influent TSS is 225 mg/L.**

Step 1 – Calculate the mass of TSS added per day

$$\text{Mass of TSS} = \text{Flow} \times \text{concentration} = 225 \text{ mg/L} \times 14.5 \text{ ML/day} = 3,262.5 \text{ kg/day}$$

Step 2 – Insert known values and solve

$$\text{Organic loading rate} = \frac{3,262.5 \text{ kg/day}}{50 \text{ m} \times 5 \text{ m}} = \frac{3,262.5 \text{ kg/day}}{250 \text{ m}^2} = 13.05 \text{ kg/m}^2/\text{day}$$

**Calculate the BOD loading rate on an aeration basin that is 30 m long, 6 m wide and 4 m deep if 4,000 cubic metres of primary effluent having a BOD of 80 mg/L is added each day.**

Step 1 – Calculate the mass of BOD added per day

$$\text{Mass of BOD} = \text{Flow} \times \text{concentration} = 80 \text{ mg/L} \times 4 \text{ ML/day} = 320 \text{ kg/day}$$

Step 2 – Calculate the volume of the aeration basin

$$\text{Volume} = \text{L} \times \text{W} \times \text{D} = 30 \text{ m} \times 6 \text{ m} \times 4 \text{ m} = 720 \text{ m}^3$$

Step 3 – Insert known values and solve

$$\text{Organic loading rate} = \frac{\text{Mass of BOD added}}{\text{Volume}} = \frac{320 \text{ kg/day}}{720 \text{ m}^3} = 0.44 \text{ kg BOD/m}^3/\text{day}$$

**Calculate the BOD loading rate on a rotating biological contactor. The influent flow is 1000 m<sup>3</sup>/day with a BOD of 185 mg/L. The RBC has 400 disks each 3.5 metres in diameter mounted on its shaft.**

The equation is:

$$\text{Organic loading rate} = \frac{\text{Flow} \times \text{concentration}}{\text{surface area} \times \text{number of disks} \times 2}$$

Step 1 – Calculate the mass of BOD applied

$$\text{Mass of BOD} = \text{Flow} \times \text{concentration} = 185 \text{ mg/L} \times 1.0 \text{ ML/day} = 185 \text{ kg/day}$$

Step 2 – Calculate surface area of disks

$$\text{Area} = \pi r^2 \times 2 \times \text{number of disks} = 3.14 \times 2 \times (1.75 \text{ m})^2 \times 400 = 7,693 \text{ m}^2$$

Step 3 – Insert known values and solve

$$\text{Organic loading rate} = \frac{\text{Mass of BOD applied}}{\text{Surface area}} = \frac{185 \text{ kg/day}}{7,693 \text{ m}^2} = 0.02 \text{ kg BOD/m}^2/\text{day}$$

*Note: organic loading to a RBC is usually reported as g BOD/m<sup>2</sup>/day.*

## Wastewater Sludge Calculations (Activated Sludge)

### Sludge Volume Index and Sludge Density Index Calculations

The sludge volume index (SVI) and sludge density index (SDI) inform the operator about the way in which activated sludge flocculates and settles in the secondary clarifier. They play a role in determining return sludge rates and mixed liquor suspended solids.

- An SVI less than 80 indicates excellent settling and compacting characteristics
- An SVI between 80 and 150 indicates moderate settling and compacting characteristics
- An SVI greater than 150 indicates poor settling and compacting characteristics

Samples for the settleability and SVI tests should be taken from the end of an actively aerated basin before clarification.

Three equations are commonly used. They are:

$$SVI = \frac{\text{Settled sludge volume, mL/L} \times 1,000 \text{ mg/g}}{\text{Mixed liquor suspended solids, mg/L}}$$

$$SVI = \frac{\text{Settled sludge volume, mL/L}}{\text{Mixed liquor suspended solids, g/L}}$$

$$SVI = \frac{\text{settled sludge volume, \%}}{\text{mixed liquor suspended solids, \%}}$$



### Sludge Volume Index (SVI)

A settleability test on an MLSS sample in a 1 litre graduated cylinder had a settled sludge volume (SSV) of 245 mL. If the MLSS concentration was 2,810 mg/L what was the sludge volume index?

Known: SSV = 245 mL, MLSS = 2,810 mg/L = 2.81 g/L

Insert known values and solve

$$SVI = \frac{\text{Settled sludge volume, mL} \times 1,000}{\text{Mixed liquor suspended solids, mg/L}} = \frac{245 \text{ mL} \times 1,000}{2,810 \text{ mg/L}} = 87 \text{ mL/g}$$

$$SVI = \frac{\text{Settled sludge volume, mL/L}}{\text{Mixed liquor suspended solids, g/L}} = \frac{245 \text{ mL/L}}{2.81 \text{ g}} = 87 \text{ mL/g}$$

The third variation of the SVI equation requires us to convert the settled sludge volume and the MLSS concentration to a per cent value.

$$SSV = \frac{245 \text{ mL}}{1,000 \text{ mL}} \times 100\% = 24.5\%$$

$$MLSS = 2,810 \frac{\text{mg}}{\text{L}} \times \frac{1\%}{10,000 \text{ mg/L}} = 0.281\%$$

Insert calculated values and solve

$$SVI = \frac{\text{settled sludge volume, \%}}{\text{mixed liquor suspended solids, \%}} = \frac{24.5\%}{0.281\%} = 87 \text{ mL/g}$$

Although the units for SVI are in mL/g the results of the calculation are usually reported as a dimensionless number.

As the examples show, all three formulas give the same answer. Operators can choose any formula but once a formula is chosen it is recommended that the operator stick with that formula to avoid confusion.

### Sludge Density Index (SDI)

The sludge density index is less commonly used. It reports a value in units of g/mL versus mL/g. (remember, density is measured as weight per unit volume)

Two formulas are available to calculate the sludge density index (SDI)

$$SDI = \frac{\text{MLSS, g} \times 100\%}{\text{Settled sludge volume, mL/L}} \text{ or } SDI = \frac{100}{\text{Sludge volume index}}$$

**A settleability test on an MLSS sample with a concentration of 2,810 mg/L carried out in a 1 litre graduated cylinder had a settled sludge volume (SSV) of 245 mL. The operator calculated that the sludge volume index (SVI) was 87. What is the sludge density index for this sample?**

Known: SSV = 245 mL, MLSS = 2,810 mg/L = 2.81 g/L

Insert known values and solve

$$\text{SDI} = \frac{\text{MLSS, g} \times 100\%}{\text{Settled sludge volume, mL/L}} = \frac{2.81 \text{ g} \times 100\%}{245 \text{ mL/L}} = 1.15 \text{ g/mL}$$

$$\text{or SDI} = \frac{100}{\text{Sludge volume index}} = \frac{100}{87} = 1.15 \text{ g/mL}$$

As with SVI the results of the calculation are usually reported as a dimensionless number.

As the examples show, both formulas give the same answer. Operators can choose either method but once a formula is chosen it is recommended that the operator stick with that formula.

### Food to Microorganism (F:M) Ratio Calculations

The food to microorganism (F:M) ratio is one of the most important calculations for the control of the activated sludge process. The operator, for all practical purposes, has no control over the volume of flow entering the plant or the concentration of BOD<sub>5</sub> contained in the flow. If he or she is to balance the food (BOD<sub>5</sub>) available to the microorganisms present to consume it the balance will be achieved by wasting or not wasting microorganisms from the process. In the F:M equation microorganisms are measured as mixed liquor volatile suspended solids (MLVSS). The F:M ratio is usually reported as a dimensionless number.

The equation is:

$$\text{Food to Microorganism ratio (F:M)} = \frac{\text{BOD}_5 \text{ added, kg}}{\text{MLVSS under aeration, kg}}$$

or

$$\text{F:M} = \frac{\text{BOD}_5 \text{ added, kg}}{\text{MLVSS concentration} \times \text{Volume of aeration basin} + \text{clarifier}}$$

Where:

$$\text{BOD}_5 \text{ added} = \text{BOD}_5, \text{mg/L} \times \text{Flow}$$

$$\text{MLVSS} = \text{Mixed liquor volatile solids, mg/L} \times \text{Volume of (aeration tank} + \text{clarifier)}$$

*Note: In most problems, the only volume or dimensions given will be for those of the aeration basin(s).*



Given the following data, calculate the F:M ratio:

Flow: 10,500 m <sup>3</sup> /day	Primary effluent BOD <sub>5</sub> = 220 mg/L
Aeration tank volume: 1,850 m <sup>3</sup>	MLVSS = 2,450 mg/L

Step 1 – Calculate the kg of BOD<sub>5</sub> added each day

$$\text{BOD}_5 \text{ added} = \frac{220 \text{ mg}}{\text{L}} \times \frac{10,500 \text{ m}^3}{\text{day}} \times \frac{1 \text{ kg}}{10^6 \text{ mg}} \times \frac{10^3 \text{ L}}{\text{m}^3} = 2,310 \text{ kg/day}$$

or

$$\text{BOD}_5 \text{ added} = 220 \text{ mg/L} \times 10.5 \text{ ML} = 2,310 \text{ kg/day}$$

Step 2 – Calculate kg of MLVSS under aeration

$$\text{MLVSS under aeration} = \frac{2,450 \text{ mg}}{\text{L}} \times \frac{1,850 \text{ m}^3}{\text{day}} \times \frac{1 \text{ kg}}{10^6 \text{ mg}} \times \frac{10^3 \text{ L}}{\text{m}^3} = 4,532.5 \text{ kg}$$

or

$$\text{MLVSS under aeration} = 2,450 \text{ mg/L} \times 1.85 \text{ ML} = 4,532.5 \text{ kg}$$

Step 3 – Insert calculated values and solve:

$$\text{F:M} = \frac{\text{BOD}_5 \text{ added, kg}}{\text{MLVSS under aeration, kg}} = \frac{2,310 \text{ kg}}{4,532.5 \text{ kg}} = 0.5$$

## Sludge Recycle Rate

The equations are

$$\text{Recycle Flow (RAS)} = \frac{\text{Flow into aeration basin} \times \text{MLSS, mg/L}}{\text{return activated sludge, mg/L} - \text{mixed liquor suspended solids, mg/L}}$$

or

$$\text{Recycle flow (RAS)} = \frac{\text{flow}}{\frac{100}{(\text{MLSS, \%} \times \text{SVI}) - 1}} \text{ or } \frac{\text{flow}}{.01 \times [(\text{MLSS, \%} \times \text{SVI}) - 1]}$$

Calculate the return activated sludge rate for a treatment plant given the following data:

Flow: 12,000 m <sup>3</sup> /day	MLSS = 2,400 mg/L
Return activated sludge: 3,600 mg/L	SVI = 212

Insert known values and solve

Equation 1

$$\text{Recycle Flow (RAS)} = \frac{\text{flow} \times \text{MLSS, mg/L}}{\text{RAS, mg/L} - \text{MLSS, mg/L}}$$

$$\text{Recycle Flow (RAS)} = \frac{12,000 \text{ m}^3/\text{d} \times 2,400 \text{ mg/L}}{3,600 \text{ mg/L} - 2,400 \text{ mg/L}} = 24,000 \text{ m}^3/\text{day}$$

Equation 2

$$\text{Recycle flow (RAS)} = \frac{\text{flow}}{.01 \times [(\text{MLSS, \%} \times \text{SVI}) - 1]}$$

$$\text{Recycle flow (RAS)} = \frac{12,000 \text{ m}^3/\text{day}}{.01 \times [(0.24\% \times 212) - 1]} = \frac{12,000 \text{ m}^3/\text{day}}{0.5} = 24,000 \text{ m}^3/\text{day}$$

### Sludge Return Rate, %

One of the parameters for the control of the activated sludge process is the rate at which settled MLSS is returned from the clarifier to the aeration basin. Different variations of the activated sludge process have different optimum return rates. In addition to the return of solids from the clarifier, some biological nutrient removal processes have internal sludge recycle streams as well.

The equation is:

$$\text{Return rate, \%} = \frac{\text{Return flow rate} \times 100}{\text{Influent flow rate}}$$

**Calculate the percent sludge return rate if the influent flow is 9,500 ML/day and the RAS return rate is 85 L/second.**

Step 1 – Calculate the RAS rate in cubic metres per day

$$\text{RAS rate} = \frac{85 \text{ L}}{\text{s}} \times \frac{86,400 \text{ s}}{\text{day}} \times \frac{1 \text{ m}^3}{1,000 \text{ L}} = 7,344 \text{ m}^3/\text{day}$$

Step 2 – Insert known values and solve:

$$\text{Return rate} = \frac{\text{Return flow rate} \times 100}{\text{Influent flow rate}} = \frac{7,344 \text{ m}^3/\text{day} \times 100}{9,500 \text{ m}^3/\text{day}} = 77\%$$

### Sludge Wasting Rate

The activated sludge process will produce between 0.4 to 0.8 kg of solids for each kg of BOD removed. In order to maintain the proper F:M ratio in the process some sludge needs to be removed or wasted from the process. Sludge wasting is normally calculated on the basis of maintaining a desired MLSS concentration in the aeration basin or on maintaining a desired MCRT. The formulas are presented below:

Wasting to maintain a **desired MLSS concentration**

$$\text{Waste sludge, m}^3 = \frac{(\text{Actual MLSS} - \text{Desired MLSS, mg/L}) \times \text{Aeration tank volume, m}^3}{\text{Return activated sludge concentration, mg/L}}$$

Wasting to maintain a **desired MCRT**

$$\text{Waste sludge, kg/day} = \frac{\text{kg MLSS in aeration basin} + \text{clarifier}}{\text{MCRT, days}} - \text{Effluent TSS, kg}$$

**A wastewater treatment plant has been found to operate best with a MLSS concentration of 2,400 mg/L. Over time the MLSS concentration has increased to 2,580 mg/L. If the plant has an aeration basin volume of 2,000 cubic metres and a RAS concentration of 3,220 mg/L how much RAS should be wasted to bring the plant back into peak performance?**

Known: Actual MLSS = 2,580 mg/L, Desired MLSS = 2,400 mg/L, RAS = 3,220 mg/L  
Aeration basin volume = 2,000 m<sup>3</sup>

Insert known values and solve

$$\text{Waste sludge, m}^3 = \frac{(\text{Actual MLSS} - \text{Desired MLSS, mg/L}) \times \text{Aeration tank volume, m}^3}{\text{Return activated sludge concentration, mg/L}}$$

$$\text{Waste sludge, m}^3 = \frac{(2,580 \text{ mg/L} - 2,400 \text{ mg/L}) \times 2,000 \text{ m}^3}{3,220 \text{ mg/L}} = 111.8 \text{ m}^3$$

**A treatment plant has been operating with a 7 day MCRT but now the operator wants to reduce the MCRT to 5 days. The MLSS concentration is 2,650 mg/L and effluent suspended solids are 8 mg/L. The combined volume of the aeration basin and clarifier is 3,582 m<sup>3</sup> and the flow through the plant is 12,500 m<sup>3</sup>/day. How many kilograms of solids need to be wasted from the process to achieve a 5 day MCRT?**

Known:

Desired MCRT = 5 days	MLSS = 2,650 mg/L	Effluent TSS = 8 mg/L
Flow = 12,500 m <sup>3</sup> /day	Aeration tank volume + Clarifier volume = 3,582 m <sup>3</sup>	

Step 1 – Calculate the kg of MLSS in inventory

$$\text{MLSS} = 2,650 \text{ mg/L} \times 3,582 \text{ ML} = 9,492 \text{ kg}$$

Step 2 – Calculate the kg of effluent TSS

$$\text{Effluent TSS} = 8 \text{ mg/L} \times 12.5 \text{ ML} = 100 \text{ kg}$$

Step 3 – Insert known and calculated values and solve

$$\text{Waste sludge, kg/day} = \frac{\text{kg MLSS in aeration basin} + \text{clarifier}}{\text{MCRT, days}} - \text{Effluent TSS, kg}$$

$$\text{Waste sludge} = \frac{9,492 \text{ kg}}{5 \text{ days}} - 100 \text{ kg} = 1,798 \text{ kg/day}$$

## Mean Cell Residence Time /Sludge Age / Solids Retention Time

Mean cell residence time, sludge age and solids retention time are all methods used by an operator to control the inventory of solids in the process and to maintain the desired food to microorganism ratio or the environment necessary for certain species to thrive (e.g. nitrifiers/denitrifiers or phosphorous accumulating microorganisms).

### Mean Cell Residence Time (MCRT)

The mean cell residence time calculation is a refinement of the solids retention (or detention) time and the sludge age calculation. MCRT takes into account solids which are stored in the secondary clarifier as well as solids that are removed from the process as waste activated sludge and effluent suspended solids. It is a subtractive process as it monitors solids lost from the process. It is an important design and operating parameter with values normally expressed in days.

The equation is:

$$\text{MCRT} = \frac{\text{MLSS, mg/L} \times (\text{volume of aeration basin} + \text{clarifier, m}^3)}{(\text{WAS, mg/L} \times \text{WAS Flow}) + (\text{Effluent TSS, mg/L} \times \text{Effluent Flow})}$$

Where: MLSS = mixed liquor suspended solids, WAS = waste activated sludge, Effluent TSS = effluent total suspended solid, Effluent flow = flow leaving the plant.

It is assumed that the solids concentration in the clarifier is the same as that in the aeration basin (i.e. the MLSS concentration)

**Given the following data, calculate the mean cell residence time for this treatment plant:**

<b>Volume of aeration basin + clarifier = 1,800 m<sup>3</sup></b>	<b>MLSS = 2,625 mg/L</b>
<b>WAS = 750 kg/day</b>	<b>Effluent TSS = 15 kg/day</b>

The simplified equation is:

$$\text{MCRT} = \frac{\text{kg solids under aeration}}{\text{kg solids lost per day}}$$

Step 1 – Calculate the kilograms of solids under aeration

$$\text{kg solids under aeration} = 2,625 \text{ mg/L} \times 1.8 \text{ ML} = 4,725 \text{ kg}$$

Step 2 – Insert known values and solve

$$\text{MCRT} = \frac{\text{kg solids under aeration}}{\text{kg solids lost per day}} = \frac{4,725 \text{ kg}}{750 \text{ kg} + 15 \text{ kg}} = 6 \text{ days}$$

### Solids Retention Time (SRT)

Like the MCRT equation, solids retention time is a subtractive process as it monitors solids lost from the process. It is slightly less accurate than the MCRT as it does not take into account solids lost in the final effluent or solids held in the secondary clarifier.

It is an important design and operating parameter with values normally expressed in days.

The equation is:

$$\text{Solids Retention Time (SRT)} = \frac{\text{MLSS, mg/L} \times \text{Aeration basin volume, m}^3}{\text{WAS, mg/L} \times \text{WAS Flow, m}^3/\text{day}}$$

**The aeration basin at a treatment plant contains 2,000 m<sup>3</sup> of MLSS with a concentration of 2,400 mg/L. The operator has set the waste rate at 325 m<sup>3</sup>/day and the WAS has a concentration of 4,800 mg/L. What is the solids retention time?**

Step 1 – Calculate the kg of MLSS under aeration

$$\text{kg MLSS} = 2,400 \text{ mg/L} \times 2.0 \text{ ML} = 4,800 \text{ kg}$$

Step 2 – Calculate the kg of WAS waste per day

$$\text{kg WAS wasted} = \frac{4,800 \text{ mg}}{\text{L}} \times \frac{325 \text{ m}^3}{\text{day}} \times \frac{1 \text{ kg}}{10^6 \text{ mg}} \times \frac{10^3 \text{ L}}{\text{m}^3} = 1,560 \text{ kg/day}$$

Step 3 – Insert known values and solve:

$$\text{SRT} = \frac{\text{kg MLSS under aeration}}{\text{kg WAS wasted/day}} = \frac{4,800 \text{ kg}}{1,560 \text{ kg/day}} = 3 \text{ days}$$

### Sludge Age

Sludge age (also known as solids retention time and Gould sludge age) is an important parameter in the operation of the activated sludge process. Similar in concept to detention time, sludge age refers to the amount of time, in days, that solids remain under aeration. It is an additive process as it measures solids added to the process each day. Sludge age is controlled by varying the waste activated sludge rate.

The equation for sludge age is:

$$\text{Sludge age, days} = \frac{\text{kg solids under aeration}}{\text{kg solids added per day}}$$

**The influent to an extended aeration package plant adds 203 kg/day of solids to the aeration basin. If the solids under aeration are 3045 kilograms, what is the sludge age in days?**

Insert known values and solve

$$\text{Sludge age, days} = \frac{\text{kg solids under aeration}}{\text{kg solids added per day}} = \frac{3,045 \text{ kg}}{203 \text{ kg/day}} = 15 \text{ days}$$

**How many kg of solids must be added to an aeration basin per day to maintain a sludge age of 20 days when 1,160 kg of solids are under aeration?**

Rearrange the equation, insert known values and solve for solids added

$$\text{Solids added} = \frac{\text{solids under aeration, kg}}{\text{Sludge age, days}} = \frac{1,160 \text{ kg}}{20 \text{ days}} = 58 \text{ kg/day}$$

## Solids under Aeration

Solids under aeration calculations are used in process control. Knowledge of the kg of solids under aeration is required to calculate the food to microorganism ratio.

**Calculate the kilograms of solids under aeration in a bioreactor with a volume of 1,100 m<sup>3</sup> and a MLSS concentration of 1,988 mg/L**

Insert known values and solve:

$$\text{Solids under aeration} = 1,988 \text{ mg/L} \times 1.1 \text{ ML} = 2,186.8 \text{ kg}$$

**An aeration basin with a volume of 1,200 m<sup>3</sup> contains 1,756 kg of MLSS, calculate the concentration of the MLSS in mg/L**

Rearrange the equation and solve for concentration

$$\text{Concentration} = \frac{\text{kg under aeration}}{\text{Volume, ML}} = \frac{1,756 \text{ kg}}{1.2 \text{ ML}} = 1,463 \text{ mg/L}$$

## Waste Activated Sludge (WAS) Pumping Rate

One of the primary control tools available to operators of the activated sludge process is wasting of excess sludge from the process in order to control the food to microorganism (F:M) ratio. Excess activated sludge may be taken from the aeration basin or from the return activated sludge line depending on the downstream processes in use. Sludge should be wasted continuously and not intermittently and by no more than 15% of the total volume from one day to the next.

**Calculate the waste activate sludge (WAS) pumping rate in liters per second if 1,452 kilograms per day are to be wasted and the WAS suspended solids concentration is 3,450 milligrams per liter.**

The question wants an answer in L/s

Step 1 – Calculate the number of milligrams wasted per second

$$\frac{1,452 \text{ kg}}{\text{day}} \times \frac{1 \text{ day}}{86,400 \text{ s}} \times \frac{10^6 \text{ mg}}{\text{kg}} = 16,800 \text{ mg/s}$$

Rearrange the standard equation to solve for volume wasted

$$\text{Volume wasted} = \frac{\text{mg wasted/sec}}{\text{concentration, mg/L}} = \frac{16,800 \text{ mg/s}}{3,450 \text{ mg/L}} = 4.9 \text{ L/s}$$

Alternate method:

$$\text{Pumping rate} = \frac{\text{WAS, kg/d}}{\text{WAS, mg/L}} = \frac{1,452 \text{ kg}}{\text{day}} \times \frac{1 \text{ L}}{3,450 \text{ mg}} \times \frac{10^6 \text{ mg}}{\text{kg}} \times \frac{1 \text{ day}}{86,400 \text{ s}} = 4.9 \text{ L/s}$$

## Slope and Grade

Wastewater treatment systems occasionally utilize gravity as a driving force to convey wastewater through pipes. Pipes need to be installed at a constant grade (or slope) to ensure that wastewater will flow at the proper velocity required to ensure that solids remain entrained in the water. Slope is expressed as a decimal value and grade is simply the slope expressed as a percentage. (i.e. a slope of 0.02 is equivalent to a grade of 2%). Solving slope and grade problems will be simplified if a sketch is drawn.

The basic equation for slope (and grade) is:

$$\text{Slope} = \frac{\text{Rise or drop}}{\text{Run}} \quad \text{and} \quad \text{Grade, \%} = \frac{\text{Rise or drop}}{\text{Run}} \times 100$$

**Calculate the slope of a pipe if it drops 2.5 meters in 90 meters.**

Known: Rise (drop) = 2.5 m, Run = 90 m

Insert known values and solve

$$\text{Slope} = \frac{\text{Rise or drop}}{\text{Run}} = \frac{2.5 \text{ m}}{90 \text{ m}} = 0.028$$

**An outfall leaves a treatment plant at an elevation 12 metres above sea level. It terminates 4.5 kilometres from the treatment plant at a depth of 80 metres below sea level. What is the grade of the outfall?**

**Known: Drop = 12m + 80 m = 92 m, Run = 4.5 km = 4,500 m**

Insert known values and solve:

$$\text{Grade, \%} = \frac{\text{Rise or drop}}{\text{Run}} \times 100 = \frac{92 \text{ m}}{4,500 \text{ m}} \times 100 = 2\%$$

## Pumping Calculations

Calculations of pump curves, required power and system heads is usually left to the design engineer. However, it is useful for the operator to be able to calculate efficiencies and capacities of pumps within his or her system in the event that a change to the system is contemplated. In the U.S. system the terms water horsepower, motor horsepower and brake horsepower are used. In the metric system the term horsepower is replaced with the term power. (1 Horsepower = 746 watts = 0.746 kW)

### Power Calculations

Two equations are used to calculate the power required to pump wastewater. They are:

**For flows in cubic metres per second:**

$$\text{Power required, kW} = \frac{9.81 \times \text{Flow, m}^3/\text{s} \times \text{Head, m} \times \text{Specific gravity}}{\text{Pump efficiency, \%}}$$

And sometimes,

$$\text{Power required, kW} = \frac{9.81 \times \text{Flow, m}^3/\text{s} \times \text{Head, m} \times \text{Specific gravity}}{\text{Pump efficiency, \%} \times \text{Motor efficiency, \%}}$$

**For flows in litres per minute**

$$\text{Power, kW} = \frac{\text{Flow, L/min} \times \text{Head, m}}{6,125}$$

**For flows in litres per second**

$$\text{Power required, kW} = \frac{9.81 \times \text{Flow, L/s} \times \text{Head, m} \times \text{Specific gravity}}{1,000 \times \text{Pump efficiency, \%} \times \text{Motor efficiency, \%}}$$

*Note: For all formulas express % as a decimal. E.g. 95% = .95*

**What is the power required in kilowatts for a pump required to meet the following parameters:**

<b>Motor efficiency = 90%</b>	<b>Pump efficiency = 85%</b>
<b>Discharge head = 45 metres</b>	<b>Flow = 5,500 m<sup>3</sup>/day</b>

Step 1 – Calculate the flow in cubic metres per second

$$\text{Flow} = \frac{5,500\text{m}^3}{\text{day}} \times \frac{1 \text{ day}}{86,400 \text{ seconds}} = 0.06 \text{ m}^3/\text{s}$$

Step 2 – Insert known values and solve

$$\text{Power required, kW} = \frac{9.81 \times \text{Flow, m}^3/\text{s} \times \text{Head, m} \times \text{Specific gravity}}{\text{Pump efficiency, \%} \times \text{Motor efficiency, \%}}$$

$$\text{Power required, kW} = \frac{9.81 \times 0.06 \text{ m}^3/\text{s} \times 45 \text{ m} \times 1}{0.9 \times 0.85} = \frac{26.487}{0.765} = 34.6 \text{ kW}$$

**Calculate the power required to pump water at a velocity of 0.4 m/sec to a height of 12 m through a 200 mm diameter pipe.**

The formula is:

$$\text{Power, kW} = \frac{\text{Flow, L/min} \times \text{Head, m}}{6,125}$$



Step 1 – Calculate the flow in L/minute

$$\text{Flow} = \text{Area} \times \text{Velocity} = 3.14 \times 0.1\text{m} \times 0.1\text{m} \times 0.4 \text{ m/s} = 0.0126 \text{ m}^3/\text{s}$$

$$\text{Flow} = \frac{0.0126\text{m}^3}{\text{s}} \times \frac{60\text{s}}{\text{minute}} \times \frac{1,000 \text{ L}}{\text{m}^3} = 756 \text{ L/minute}$$

Step 2 – Insert known values and solve

$$\text{Power, kW} = \frac{\text{Flow, L/min} \times \text{Head, m}}{6,125} = \frac{756 \text{ L/min} \times 12 \text{ m}}{6,125} = 1.5 \text{ kW}$$

### Efficiency Calculations

Before discussing the formulas and calculations surrounding the efficiency of a pump, motor and pump-motor combination it will be useful to first define some terms.

- Motor Horsepower (mhp) is a measure of the electrical power supplied to the terminals of the electric motor. It is the input power to the motor. One horsepower is defined as being equal to 746 Watts or 0.746 kilowatt.
- Brake Horsepower (bhp) is the output power of the motor. It is also known as the shaft horsepower (shp). The brake horsepower of a motor is always less than the input or motor horsepower supplied to the motor due to friction, resistance within the stator, rotor and core and the load applied to the motor.
- Water Horsepower (whp) is the output power of a pump. That is, the energy imparted to the fluid being pumped in order to raise a given volume of it to a given height. The water horsepower is always less than the shaft or brake horsepower applied to the pump shaft due to friction, friction losses and inefficiencies in impellor and volute design.
- Wire to water horsepower (also called wire-to-water efficiency or overall efficiency) is the energy that is imparted to the water divided by the energy supplied to the motor. It is work done divided by work applied.

The term metric horsepower is strictly defined as the power required to raise a mass of 75 kilograms against the earth's gravitational force over a distance of one metre in one second; this is equivalent to 735.49875 Watts or 98.6% of an imperial electrical horsepower which is equal to 746 Watts.

In this manual and in the EOCP and ABC handouts 1 horsepower = 746 Watts.

The formulas used to measure efficiency in pumping applications are:

$$\text{Motor efficiency} = \frac{\text{Brake horsepower} \times 100}{\text{Motor horsepower}} \text{ or } \frac{\text{bhp} \times 100}{\text{mhp}}$$

$$\text{Pump efficiency} = \frac{\text{Water horsepower} \times 100}{\text{Brake horsepower}} \text{ or } \frac{\text{whp} \times 100}{\text{bhp}}$$

$$\text{Overall efficiency (wire to water efficiency)} = \frac{\text{Water horsepower} \times 100}{\text{Motor horsepower}} \text{ or } \frac{\text{whp} \times 100}{\text{mhp}}$$

or

$$\text{Wire to water efficiency} = \text{Decimal motor efficiency} \times \text{decimal pump efficiency} \times 100\%$$

**What is the motor power if the brake power is 35 kW and the motor efficiency is 88%?**

Insert known values and solve

$$\text{Motor horsepower} = \frac{\text{Brake horsepower} \times 100}{\text{Motor efficiency, \%}} = \frac{40 \text{ kW} \times 100}{88\%} = 45.5 \text{ kW}$$

**Find the water horsepower if the brake horsepower is 34 kW and the pump efficiency is 81%**

The equation is: Water horsepower = (brake horsepower)(pump efficiency)

$$\text{Water horsepower} = (\text{brake horsepower})(\text{pump efficiency}) = (34 \text{ kW})(0.81) = 27.5 \text{ kW}$$

**What is the brake horsepower if the water horsepower is 40 kW and the pump efficiency is 78%?**

Step 1 – Rearrange the water horsepower equation, insert known values and solve

$$\text{Brake horsepower} = \frac{\text{water horsepower}}{\text{efficiency}} = \frac{40 \text{ kW}}{.78} = 51 \text{ kW}$$

**What is the motor horsepower if 60 kW of water horsepower is required to run a pump with a motor efficiency of 93% and a pump efficiency of 85%?**

The equation is:

$$\text{Motor horsepower} = \frac{\text{water horsepower}}{\text{motor efficiency} \times \text{pump efficiency}}$$

Insert known values and solve

$$\text{Motor horsepower} = \frac{60 \text{ kW}}{.93 \times .85} = \frac{60 \text{ kW}}{.79} = 76 \text{ kW}$$

## PART 2 – FORMULAS AND PRACTICE QUESTIONS – ABC HANDOUT

This section of the manual contains additional formulas which are not included in the EOCP handout for wastewater treatment. Each formula is followed by a sample question which illustrates how the formula is to be used. Some of the formulas in the ABC handout have already been covered in the previous section as they duplicate formulas contained in the EOCP handout.

### Basic Electricity

The Law which relates voltage, amperage and resistance in an electrical circuit known as Ohm's Law. Ohm's Law states that the electromotive force (voltage) in a circuit is the product of current flow (amperes) and resistance (ohms).

Four formulas can be derived from Ohm's Law:

$$E = I \times R \quad \text{or} \quad I = \frac{E}{R} \quad \text{or} \quad R = \frac{E}{I} \quad \text{and} \quad P = E \times I$$

Where: E = Volts, I = Amperes, R = resistance measured in Ohms, P = power measured in Watts

Three different formula are used to calculate power depending on whether the voltage supply is direct current, alternating current or three-phase alternating current.

The formulas are:

$$\text{Power, kW in a Direct current circuit} = \frac{\text{Volts} \times \text{Amperes}}{1,000}$$

$$\text{Power, kW in an Alternating current circuit} = \frac{\text{Volts} \times \text{Amperes} \times \text{Power Factor}}{1,000}$$

$$\text{Power, kW in an Alternating } 3\phi \text{ current circuit} = \frac{\text{Volts} \times \text{Amperes} \times \text{Power Factor} \times 1.732}{1,000}$$

Operators should have a basic knowledge of the application of Ohm's Law so as to undertake simple electrical calculations.

**What is the voltage (E) on a circuit if the current (I) is 7 amperes and the resistance (R) is 17 ohms? Give the answer to 3 significant figures.**

The equations is:  $E = (I)(R)$

Insert known values and solve

$$\text{Voltage} = (7 \text{ amps})(17 \text{ ohms}) = 119 \text{ Volts}$$

**What is the resistance in a circuit if the voltage is 120 and the amperes are 19?**

The equation is:  $R = E / I$

Insert known values and solve

$$\text{Resistance} = (120 \text{ volts}) / (19 \text{ amperes}) = 6.3 \text{ ohms}$$

## Biochemical Oxygen Demand (BOD<sub>5</sub>) – Unseeded and Seeded

Biochemical oxygen demand measures the amount of oxygen consumed by microorganisms as they metabolize organic material – either in a wastewater treatment process or in the natural environment.

The test is carried out in a darkened incubator over five days  $\pm 6$  hours at a temperature of  $20^\circ\text{C} \pm 1^\circ\text{C}$  in a 300 mL BOD bottle. The acronym used is often written as BOD<sub>5</sub>. BOD<sub>5</sub> values are typically expressed in mg/L.

The formula for calculating an unseeded BOD sample is:

$$\text{BOD, mg/L} = \frac{(\text{DO}_{\text{initial}} - \text{DO}_{\text{final}}) \times 300 \text{ mL}}{\text{Sample volume, mL}}$$

Occasionally an operator will need to calculate the BOD of a sample which has been disinfected and contains no viable microorganisms. In this case the sample needs to be “seeded” with a small aliquot of wastewater with a known BOD concentration. Two calculations are needed in this case.

The formula for calculating the seed correction in mg/L is:

$$\text{Seed correction, mg/L} = \frac{\text{BOD of seed stock, mg/L} \times \text{Volume of seed stock, mL}}{\text{Total Volume of BOD bottle, 300 mL}}$$

The formula for calculating the seeded BOD is:

$$\text{BOD, mg/L} = \frac{(\text{DO}_{\text{initial}} - \text{DO}_{\text{final}} - \text{Seed correction}) \times 300 \text{ mL}}{\text{Sample volume, mL}}$$

Where  $\text{DO}_{\text{initial}}$ ,  $\text{DO}_{\text{final}}$  and Seed Correction are all expressed in mg/L

**A 25 mL sample of final effluent had an initial DO of 6.2 mg/L and a final DO of 3.9 mg/L. Calculate the BOD of the sample.**

Known:  $\text{DO}_{\text{initial}} = 6.2 \text{ mg/L}$ ,  $\text{DO}_{\text{final}} = 3.9 \text{ mg/L}$ , Sample volume = 25 mL

Insert known values and solve:

$$\text{BOD} = \frac{(\text{DO}_{\text{initial}} - \text{DO}_{\text{final}}) \times 300 \text{ mL}}{\text{Sample volume, mL}} = \frac{(6.2 \text{ mg/L} - 3.9 \text{ mg/L}) \times 300 \text{ mL}}{25 \text{ mL}} = 27.6 \text{ mg/L}$$

**Calculate the seeded BOD<sub>5</sub> in mg/L given the following data:**

$\text{DO}_{\text{initial}}$ : 8.6 mg/L	$\text{DO}_{\text{final}}$ : 3.2 mg/L	Sample size: 125 mL
Seed stock sample: 5 mL	Seed stock BOD: 95 mg/L	Total diluted volume: 300 mL

Insert known values and solve:

Step 1 – Calculate the seed correction in mg/L

$$\text{Seed correction, mg/L} = \frac{\text{BOD of seed stock, mg/L} \times \text{Volume of seed stock, mL}}{\text{Total Volume of BOD bottle, 300 mL}}$$

$$\text{Seed correction} = \frac{95 \text{ mg/L} \times 5 \text{ mL}}{300 \text{ mL}} = 1.58 \text{ mg/L}$$

Step 2 – Calculate the seeded BOD

$$\text{BOD, mg/L} = \frac{(\text{DO}_{\text{initial}} - \text{DO}_{\text{final}} - \text{Seed correction}) \times 300 \text{ mL}}{\text{Sample volume, mL}}$$

$$\text{BOD, mg/L} = \frac{(8.6 \text{ mg/L} - 3.2 \text{ mg/L} - 1.58 \text{ mg/L}) \times 300 \text{ mL}}{125 \text{ mL}} = 9.2 \text{ mg/L}$$

### Chemical Feed Pump Setting - % Stroke

Many chemical feed pumps have the ability to vary their output by changing the length of the stroke, the frequency of the stroke or both. Adjustments are made to ensure that the optimum chemical dosage is applied.

The formula is:

$$\text{Feed pump setting, \% stroke} = \frac{\text{Desired output/unit of time} \times 100\%}{\text{Maximum output/unit of time}}$$

**A diaphragm pump used to meter sodium hypochlorite has a maximum output of 10 L/s. What % stroke should be selected to deliver 2.3 L/s**

$$\text{Feed pump setting, \% stroke} = \frac{\text{Desired output, L/s} \times 100\%}{\text{Maximum output, L/s}} = \frac{2.3 \text{ L/s} \times 100\%}{10 \text{ L/s}} = 23\%$$

### Composite Sample

When sampling, it is important that the size of sample taken is representative of the whole. Grab samples are taken to get an instantaneous snapshot of the process while composite samples are taken to get a picture of the process over a longer time period.

The equation for selecting a single sample size is:

$$\text{Composite sample single portion} = \frac{\text{instantaneous flow} \times \text{total sample volume}}{\text{number of samples} \times \text{average flow}}$$

**A treatment plant uses a composite sample to sample for TSS in the influent. The sampler is set to take 24 samples over the course of 24 hours for a total sample volume of 10 litres. The daily flow through the plant is 12,500 cubic metres. Calculate the sample volume that would be taken at a time when the instantaneous flow was 870 m<sup>3</sup>/hour.**

Insert known values and solve:

$$\text{Composite sample single portion} = \frac{870 \text{ m}^3/\text{hr} \times 10 \text{ L}}{24 \times 520.8 \text{ m}^3/\text{hr}} = 0.69 \text{ L}$$

## Cycle Time – Pumping

Establishment of appropriate pumping cycle times is important to protect the life of electrical motors and to prevent the development of septic conditions in wet wells, clarifiers and sumps.

The equation is:

$$\text{Cycle time, minutes} = \frac{\text{Wet well storage volume, m}^3}{\text{Pump capacity, m}^3/\text{minute} - \text{Wet well inflow, m}^3/\text{minute}}$$

**Calculate the cycle time for a wet well that is 3 m in diameter and 3.5 m deep if the inflow to the wet well is 0.55 m<sup>3</sup>/minute and the lift pump has a capacity of 30 L/s**

Step 1 – Calculate the volume of the wet well

$$\text{Volume} = \pi r^2 h = 3.14 \times 1.5\text{m} \times 1.5\text{m} \times 3.5\text{m} = 24.7\text{m}^3$$

Step 2 – Convert pump capacity to m<sup>3</sup>/minute

$$\text{Pump output} = \frac{30 \text{ L}}{\text{s}} \times \frac{60 \text{ s}}{\text{min}} \times \frac{1 \text{ m}^3}{1,000 \text{ L}} = 1.8 \text{ m}^3/\text{minute}$$

Step 3 – Insert known values and solve

$$\text{Cycle time} = \frac{24.7 \text{ m}^3}{1.8 \text{ m}^3/\text{minute} - 0.55 \text{ m}^3/\text{minute}} = \frac{24.7 \text{ m}^3}{1.25 \text{ m}^3/\text{minute}} = 19.8 \text{ minutes}$$

## Filtration Rate Calculations

Wastewater treatment plants are using filtration to achieve effluents which meet increasingly stringent requirements for low levels of TSS and BOD. Regardless of whether the filter uses sand or a synthetic media, backwashing is required to clean the filter.

The equations typically used are:

$$\text{Filtration rate} = \frac{\text{Flow}}{\text{Volume}} \text{ or } \frac{\text{Flow}}{\text{Area}}$$

$$\text{Filter backwash rate, L/m}^2/\text{s} = \frac{\text{Flow, L/s}}{\text{Filter area, m}^2}$$

$$\text{Filter backwash rise rate, cm/minute} = \frac{\text{Water rise, cm}}{\text{Time, minutes}}$$

And for solids production on a vacuum coil filter:

$$\text{Filter yield, kg/m}^2/\text{hour} = \frac{\text{Solids concentration, \%} \times \text{sludge feed rate, L/hr} \times 10}{\text{Surface area of filter, m}^2}$$

### Filter Flow and Filter Backwash Rate

Filters are backwashed to release the impurities trapped in the filter media. Backwashing may be initiated on a timed cycle or on differential head.

The equations are the same for both forward and backwash flow, only the direction the water travels is different.

The equation is:

$$\text{Filter backwash rate, L/m}^2/\text{s} = \frac{\text{Flow, L/s}}{\text{Filter area, m}^2}$$

**A filter having a surface area of 3 m by 5 m is backwashed at a rate of 20 L/s for 1 minute. What is the filter backwash rate?**

Insert known values and solve

$$\text{Filter backwash rate, L/m}^2/\text{s} = \frac{\text{Flow, L/s}}{\text{Filter area, m}^2} = \frac{20 \text{ L/s}}{3\text{m} \times 5\text{m}} = 1.3 \text{ L/m}^2/\text{s}$$

### Filter Backwash Rise Rate

**A filter that is 3.5 m long by 6 m wide is backwashed at a rate of 205 L/s for 1 minute. What is the filter backwash rise rate?**

Step 1 – Calculate the surface area of the filter.

$$\text{Surface area} = 6\text{m} \times 3.5\text{m} = 21\text{m}^2$$

Step 2 – Calculate the backwash rate in cubic metres

$$\text{Backwash rate} = \frac{205 \text{ L}}{\text{s}} \times \frac{1 \text{ m}^3}{1,000 \text{ L}} \times \frac{60 \text{ s}}{\text{minute}} = 12.3 \text{ m}^3/\text{minute}$$

Step 3 – Calculate the depth (rise) of the backwash water

$$\text{Depth} = \frac{\text{Volume}}{\text{Area}} = \frac{12.3\text{m}^3}{21 \text{ m}^2} = 0.58\text{m} = 58 \text{ cm}$$

Insert calculated values and solve:

$$\text{Filter backwash rise rate, cm/minute} = \frac{\text{Water rise, cm}}{\text{Time, minutes}} = \frac{58 \text{ cm}}{1 \text{ minute}} = 58 \text{ cm/minute}$$

### Vacuum Coil Filter Yield

The filter yield equation is used in the operation of vacuum filter units. Vacuum filters have for the most part, been replaced by either belt filter presses, rotary presses or centrifuges.

The equation is:

$$\text{Filter yield, kg/m}^2/\text{hour} = \frac{\text{Solids concentration, \%} \times \text{sludge feed rate, L/hr} \times 10}{\text{Surface area of filter, m}^2}$$

**A rotary drum vacuum filter fed at a rate of 4.5 L/s produces a cake with 25% solids content. The drum has a surface area of 28 square metres. What is the filter yield?**

Step 1 – Convert L/s to L/hour

$$\text{Sludge feed rate} = \frac{4.5 \text{ L}}{\text{s}} \times \frac{3,600 \text{ s}}{\text{hour}} = 16,200 \text{ L/hour}$$

Step 2 – Convert 25% to a decimal value, insert known values and solve:

$$\text{Filter yield, kg/m}^2/\text{hour} = \frac{\text{Solids concentration, \%} \times \text{sludge feed rate, L/hr} \times 10}{\text{Surface area of filter, m}^2}$$

$$\text{Filter yield, kg/m}^2/\text{hour} = \frac{.25 \times 16,200 \text{ L/hr} \times 10}{28 \text{ m}^2} = 1,446 \text{ kg/m}^2/\text{hour}$$

**Four filters have a surface area of 50 m<sup>2</sup> each. What is the filtration rate in litres per second (L/s) if they receive a total flow of 0.85m<sup>3</sup>/s?**

Step 1 – Calculate the total surface area of the four filters

$$\text{Total surface area} = 4 \times 50\text{m}^2 = 200 \text{ m}^2$$

Step 2 – Convert flow from m<sup>3</sup>/s to L/s

$$\text{Flow} = \frac{0.85\text{m}^3}{\text{s}} \times \frac{1,000 \text{ L}}{\text{m}^3} = 850 \text{ L/s}$$

Step 3 - Calculate the filtration rate.

$$\text{Flow} = \frac{\text{Flow}}{\text{Area}} = \frac{850 \text{ L/s}}{200 \text{ m}^2} = 4.25 \text{ L/s/m}^2$$

## Force and Pressure

Pressure is a measure of a force against a surface and is usually expressed as force per unit area. In the metric system pressure is measured and expressed in Pascals (Pa) or kilopascals (kPa). Force is measured in Newtons (N) or kiloNewtons (kN)

One Pascal is equal to a force of one Newton per square metre. A Newton is equal to the force required to accelerate one kilogram at a rate of one metre per second per second (1kgm/s<sup>2</sup>)

A column of water 1 metre high exerts a pressure of 9.804139432 kPa. This manual will use a rounded value of 9.8 kPa

Atmospheric pressure at sea level is 101.325 kPa

The equation is:

$$\text{Force, N} = \text{Pressure, Pa} \times \text{Area, m}^2$$

$$\text{Head, m} = \text{Pressure, kPa} \times \frac{1\text{m}}{9.8 \text{ kPa}}$$



**What is the depth of water in a storage tank if the pressure at the bottom of the tank is 54 kPa?**

Insert known values and solve:

$$\text{Head} = \text{Pressure, kPa} \times \frac{1\text{m}}{9.8 \text{ kPa}} = 54 \text{ kPa} \times \frac{1\text{m}}{9.8 \text{ kPa}} = 5.5 \text{ m}$$

**What pressure will a pump generate if it can lift water to a height of 42 meters? (Assume no friction losses in the piping system)**

Rearrange the equation, insert known values and solve:

$$\text{Pressure} = \text{Head, m} \times \frac{9.8 \text{ kPa}}{\text{m}} = 42 \text{ m} \times \frac{9.8 \text{ kPa}}{\text{m}} = 411.6 \text{ kPa}$$

**A 250 mm diameter pipeline is pressurized to 750 kPa. What is the force in Newton on an end cap on the pipe?**

Known: Diameter = 250 mm = 0.25 m, radius = .25 m ÷ 2 = 0.125 m, 1kPa = 1,000 Pa

The equation is:

$$\text{Force, N} = \text{Pressure, Pa} \times \text{Area, m}^2$$

Step 1 – Calculate the surface area of the end cap

$$\text{Area} = \pi r^2 = 3.14 \times 0.125\text{m} \times 0.125\text{m} = 0.049\text{m}^2$$

Insert known values and solve

$$\text{Force} = \text{Pressure} \times \text{Area} = 750\text{kPa} \frac{1,000 \text{ Pa}}{\text{kPa}} \times 0.049\text{m}^2 = 36,796 \text{ Newtons}$$

## Basic Chemistry

### Number of Moles

A general discussion of the periodic table of the elements, Avogadro's number and the derivation of an element's atomic weight is beyond the scope of this manual. In simplest terms, a mole of a substance is a quantity of that substance whose mass (weight) in grams is equal to its atomic weight or the sum of atomic weights of the elements which make up a molecule.

For example, carbon has an atomic weight of 12 and therefore, one mole of carbon weighs 12 grams. Water (H<sub>2</sub>O) contains two atoms of hydrogen each of which have an atomic weight of 1 and one atom of oxygen which has an atomic weight of 16 for a total atomic weight of 18 and therefore, 1 mole of water weighs 18 grams.

In the chemistry laboratory we often need to know how many moles of a substance are present.

The equation is:

$$\text{Number of moles} = \frac{\text{Total weight, g}}{\text{Molecular weight, g}}$$

Calculate the number of moles of calcium hydroxide that are present in a 25 gram sample of the material. The atomic weights are: calcium = 40, oxygen = 16 and hydrogen = 1

Step 1 – Calculate the gram molecular weight of calcium hydroxide  $\text{Ca}(\text{OH})_2$

$$\text{Ca} = 1 \times 40 = 40 \quad \text{O} = 2 \times 16 = 32 \quad \text{H} = 2 \times 1 = 2 = 40 + 32 + 2 = 74 \text{ grams}$$

Insert known values and solve

$$\text{Number of moles} = \frac{\text{Total weight, g}}{\text{Molecular weight, g}} = \frac{25 \text{ g}}{74 \text{ g}} = 0.34 \text{ moles}$$

### Number of Equivalent Weights

Equivalent weight (also known as gram equivalent) is a term which has been used in several contexts in chemistry. In its most general usage, it is the mass of a given substance (mass of one equivalent) which will:

- combine or displace directly or indirectly with 1.008 parts by mass of hydrogen or 8 parts by mass of oxygen.– values which correspond to the atomic weight divided by the usual valence,
- or supply or react with one mole of hydrogen cations ( $\text{H}^+$ ) in an acid–base reaction,
- or supply or react with one mole of electrons ( $\text{e}^-$ ) in a redox reaction.

Equivalent weights have the dimensions and units of mass, unlike atomic weight, which is dimensionless. The equivalent weight of a compound can be calculated by dividing the molecular weight by the number of positive or negative electrical charges that result from the dissolution of the compound.

The use of equivalent weights in general chemistry has largely been superseded by the use of molar masses. Equivalent weights may be calculated from molar masses if the chemistry of the substance is well known. For example:

- sulfuric acid has a molar mass of 98.078 g/mol, and supplies two moles of hydrogen ions per mole of sulfuric acid, so its equivalent weight is  $98.078 \text{ g/mol} \div 2 \text{ eq/mol} = 49.039 \text{ g/eq}$ .
- potassium permanganate has a molar mass of 158.034 g/mol, and reacts with five moles of electrons per mole of potassium permanganate, so its equivalent weight is  $158.034 \text{ g/mol} \div 5 \text{ eq/mol} = 31.6068 \text{ g/eq}$

The equation is:

$$\text{Number of equivalent weights} = \frac{\text{total weight, g}}{\text{equivalent weight, g}}$$

**If 75 g of sulfuric acid ( $\text{H}_2\text{SO}_4$ ) were used in making up a solution, how many equivalents weights of  $\text{H}_2\text{SO}_4$  were used?**

Step 1 – Calculate the equivalent weight of  $\text{H}_2\text{SO}_4$

$$\text{Equivalent weight} = \frac{\text{weight of 1 mole H}_2\text{SO}_4}{\text{equivalents per mole}} = \frac{98.09 \text{ g}}{2} = 49.04 \text{ g}$$

Step 2 – Insert known values and solve

$$\text{Number of equivalent weights} = \frac{\text{total weight, g}}{\text{equivalent weight, g}} = \frac{75 \text{ g}}{49.04 \text{ g}} = 1.53$$

### Molarity

Molarity (also called molar concentration, amount concentration or substance concentration) is a measure of the concentration of a solute in a solution in terms of the amount of a substance in a given volume.

In the International System of Units (SI) the base unit for molar concentration is mol/m<sup>3</sup>. However, this is impractical for most laboratory purposes and most chemical literature traditionally uses mol/L. A solution of concentration 1 mol/L is also denoted as 1 molar (1 M).

The equation is:

$$\text{Molarity} = \frac{\text{Moles of Solute}}{\text{Litres of Solution}}$$

**What is the molarity of a solution that contains 35 grams of sodium chloride (NaCl) in 2.5 litres of water? The gram molecular weight of sodium is 22.99 g and the gram molecular weight of chlorine is 35.54 g**

Known: weight of NaCl=35 g, Weight of 1 Mole of NaCl = 22.99 g+35.54 g = 58.53 g

Step 1 – Calculate the number of moles of NaCl

$$\text{Moles NaCl} = \frac{35 \text{ g}}{58.53 \text{ g}} = 0.59 \text{ moles}$$

Step 2 – Insert known and calculated values and solve:

$$\text{Molarity} = \frac{\text{Moles of Solute}}{\text{Litres of Solution}} = \frac{0.59 \text{ moles}}{2.5 \text{ L}} = 0.24 \text{ mol/L}$$

### Normality

Normality is defined as the number of equivalent weights of a solute per litre of solution. In order to determine the normality of a solution one must first calculate how many equivalent weights of the solute are contained in the total weight of the solution.

When carrying out an acid-base titration, the number of hydrogen atoms in an acid molecule can provide a quick indication of the normality of an acid which contains one mole per litre. For example

- Hydrochloric acid (HCl) contains one hydrogen atom and if the concentration of the acid were 1 mole / litre its normality would be 1
- Sulfuric acid (H<sub>2</sub>SO<sub>4</sub>) contains two hydrogen atoms and if the concentration of the acid were 1 mole / litre its normality would be 2
- Phosphoric acid (H<sub>3</sub>PO<sub>4</sub>) contains three hydrogen atoms and if the concentration of the acid were 1 mole / litre its normality would be 3

The equation is:

$$\text{Normality} = \frac{\text{number of equivalent weights of solute}}{\text{litres of solution}}$$

**If 2.1 equivalents of sodium hydroxide (NaOH) were used to make 1.75 litres of solution what is the normality of the solution?**

Step 1 – Insert known values and solve:

$$\text{Normality} = \frac{\text{number of equivalent weights of solute}}{\text{litres of solution}} = \frac{2.1 \text{ eq}}{1.75 \text{ L}} = 1.2 \text{ N}$$

*Note: Operators who wish to delve a little deeper into the field of Chemistry may wish to obtain copies of the American Water Works Association (AWWA) publications Basic Chemistry for Water and Wastewater Operators (ISBN 1-58321-148-9) by D.S. Sarai, PhD. or Basic Science Concepts and Applications for Wastewater (ISBN 1-58321-290-6) by P.L. Antonelli et al*

### Alkalinity

The alkalinity of a wastewater is a measure of its ability to resist changes in pH. It is measured as mg/L of calcium carbonate (CaCO<sub>3</sub>).

The formula is:

$$\text{Alkalinity as mg CaCO}_3/\text{L} = \frac{\text{titrant volume, mL} \times \text{acid normality} \times 50,000}{\text{sample volume, mL}}$$

**A 100 mL sample of effluent was titrated with 22 mL of 0.2N sulfuric acid. What was its alkalinity?**

$$\text{Alkalinity} = \frac{22 \text{ mL} \times 0.02 \times 50,000}{100 \text{ mL}} = 96.8 \text{ mg/L as CaCO}_3$$

### Hardness

The hardness of a water is normally of more interest to water treatment operators than to wastewater operators. However, hard water can lead to scaling in boilers and heat exchanger piping.

When the titration factor is 1.00 of EDTA, the formula is:

$$\text{Hardness, as mg/L CaCO}_3 = \frac{\text{Titrant volume, mL} \times 1,000}{\text{Sample volume, mL}}$$

When the titration factor is some number other than 1.00 of EDTA the formula is:

$$\text{Hardness (EDTA), as mg/L CaCO}_3 = \frac{\text{Titrant volume, mL} \times \text{mg CaCO}_3 \text{ equivalent to 1 mL EDTA titrant} \times 1,000}{\text{Sample volume, mL}}$$

**What is the CaCO<sub>3</sub> hardness of a water sample if 42 mL of titrant is required to reach the endpoint (where the colour changes from wine red to blue) on a 100 mL sample?**

Known: titrant volume = 42 mL, sample volume = 100 mL

Insert known values and solve

$$\text{Hardness, as mg/L CaCO}_3 = \frac{\text{Titrant volume, mL} \times 1,000}{\text{Sample volume, mL}}$$

$$\text{Hardness} = \frac{42 \text{ mL} \times 1,000}{100 \text{ mL}} = 420 \text{ mg/L as CaCO}_3$$

### Oxygen Uptake Rate (OUR)

The Oxygen Uptake Rate (OUR) test measures the amount of oxygen consumed by a sample over a period of time. It is measured in mg/L O<sub>2</sub>/minute or mg/L O<sub>2</sub>/hour.

The equation is:

$$\text{Oxygen uptake rate} = \frac{\text{initial DO, mg/L} - \text{final DO, mg/L}}{\text{elapsed time, minutes}}$$

These quick tests have many advantages; rapid measure of influent organic load and biodegradability, indication of the presence of toxic or inhibitory wastes, degree of stability and condition of a sample, and calculation of oxygen demand rates at various points in the aeration basin. As always, trends are more useful than instantaneous values.

**Calculate the OUR of a sample if the initial dissolved oxygen concentration is 5.9 mg/L and after 10 minutes the final dissolved oxygen concentration is 1.4 mg/L.**

Insert known values and solve

$$\text{OUR} = \frac{5.9 \text{ mg/L} - 1.4 \text{ mg/L}}{10 \text{ minutes}} \times \frac{60 \text{ minutes}}{\text{hour}} = 27 \text{ mg/L O}_2/\text{hour}$$

### Specific Oxygen Uptake Rate (SOUR)

The Specific Oxygen Uptake Rate (SOUR), also known as the oxygen consumption or respiration rate, is defined as the milligrams of oxygen consumed per gram of volatile suspended solids (VSS) per hour.

The equation is:

$$\text{SOUR} = \frac{\text{Initial DO, mg/L} - \text{Final DO, mg/L}}{\text{elapsed time, minutes}} \times \frac{60 \text{ minutes}}{\text{hour}} \times \frac{1,000 \text{ mg/g}}{\text{MLVSS, mg/L}}$$

**Calculate the specific oxygen uptake rate (SOUR) of a sample with a volatile solids concentration of 2,400 mg/L if the initial dissolved oxygen concentration was 4.4 mg/L and the final dissolved oxygen concentration was 2.1 mg/L after 10 minutes**

Insert known values and solve:

$$\text{SOUR} = \frac{\text{Initial DO, mg/L} - \text{Final DO, mg/L}}{\text{elapsed time, minutes}} \times \frac{60 \text{ minutes}}{\text{hour}} \times \frac{1,000 \text{ mg/g}}{\text{MLVSS, mg/L}}$$

$$\text{SOUR} = \frac{4.4 \text{ mg/L} - 2.1 \text{ mg/L}}{10 \text{ minutes}} \times \frac{60 \text{ minutes}}{\text{hour}} \times \frac{1,000 \text{ mg/g}}{2,400 \text{ mg/L}} = 5.75 \text{ mg/O}_2/\text{g MLVSS}/\text{hour}$$

## Solids Concentration

The ability to calculate the solids concentration of a sample is a skill which every treatment plant operator who works in the laboratory must develop. Knowledge of the amount of solids entering, leaving and within a plant is essential for process control and permit compliance.

The formula is:

$$\text{Solids, mg/L} = \frac{\text{Weight of dry solids, g} \times 1,000,000}{\text{Sample volume, mL}}$$

A 25 mL sample was filtered on a Whatman GF/C 5.5 cm diameter filter. The weight of the filter paper was 0.1785 grams and the weight of the dried filter paper plus retained solids was 0.1833 grams. What was the solids concentration for this sample?

Step 1 – Calculate the weight of dry solids

$$\text{Dry solids} = 0.1833 \text{ g} - 0.1785 \text{ g} = 0.0048 \text{ g}$$

Step 2 – Insert calculate value and solve:

$$\text{Solids} = \frac{0.0048 \text{ g} \times 1,000,000}{25 \text{ mL}} = 192 \text{ mg/L}$$

## Population Equivalent Calculations

Knowledge of typical per capita water usage or BOD<sub>5</sub> contributions can be used to calculate the population load on a wastewater treatment plant or conversely, if the population is known to determine whether excessive infiltration and inflow is present.

The general equations are:

$$\text{Population equivalent} = \frac{\text{Population served}}{\text{Size of treatment process (e. g. area or volume)}}$$

$$\text{Population equivalent (organic loading)} = \frac{\text{Flow, m}^3/\text{day} \times \text{BOD, mg/L}}{1,000 \times 0.077 \text{ kg BOD/person/day}}$$

**Calculate the population loading on a lagoon if the population is 12,500 people and the lagoon system totals 8 hectares**

$$\text{Population equivalent} = \frac{\text{Population served}}{\text{Area, ha}} = \frac{12,500}{8 \text{ ha}} = 1,562 \text{ people/ha}$$

**A treatment plant receives a daily flow of 9,500 m<sup>3</sup> with a BOD of 222 mg/L. Calculate the equivalent population served.**

The equation is:

$$\text{Population equivalent (organic loading)} = \frac{\text{Flow, m}^3/\text{day} \times \text{BOD, mg/L}}{1,000 \times 0.077 \text{ kg BOD/person/day}}$$

Step 1 – Insert known values and solve

$$\text{Population equivalent} = \frac{9,500 \text{ m}^3/\text{day} \times 222 \text{ mg/L}}{1,000 \times 0.077 \text{ kg BOD/person/day}} = \frac{2,109,000}{77} = 27,389 \text{ people}$$

Alternate method:

Step 1 – Calculate the kg of BOD<sub>5</sub> received at the plant each day

$$\text{kg BOD} = \text{flow} \times \text{concentration} = 9.5 \text{ ML} \times 222 \text{ mg/L} = 2,109 \text{ kg/day}$$

Step 2 – Insert known values and solve

$$\text{Population} = \frac{\text{kg BOD /day}}{0.077 \text{ kg BOD/ person/day}} = \frac{2,109 \text{ kg BOD}}{0.077 \text{ kg BOD/ person/day}} = 27,389 \text{ people}$$

### Recirculation Ratio – Trickling Filter

Recirculation of flow from the secondary clarifier to the trickling filter is a technique used to dilute the strength of the influent to the trickling filter, maintain a relatively uniform flow to the filter, reduce odor and filter flies and to ensure the filter does not dry out during periods of low flow. Recirculation ratios generally range from 1:1 to 2:1

The equation is:

$$\text{Recirculation ratio} = \frac{\text{recirculated flow}}{\text{primary effluent flow}}$$

**What is the recirculation ratio for a trickling filter if the influent to the plant is 12.5 ML/day and a flow of 21.8 ML/day is recirculated to the trickling filter?**

Known: Primary effluent flow = 12.5 ML/day, Recirculated flow = 21.8 ML/day

Insert known values and solve:

$$\text{Recirculation ratio} = \frac{\text{recirculated flow}}{\text{primary effluent flow}} = \frac{21.8}{12.5} = 1.74:1$$

**What is the trickling filter's recirculated flow if the influent flow to the plant was 5.9 ML/day and the recirculation ratio was 1.65:1?**

Rearrange the equation to solve for recirculated flow then insert known values and solve

$$\text{If Recirculation ratio} = \frac{\text{recirculated flow}}{\text{primary effluent flow}}$$

Then Recirculated flow = Recirculation ratio × Primary effluent flow

$$\text{Recirculated flow} = 1.65 \times 5.9 \text{ ML/day} = 9.74 \text{ ML/day}$$

## Density and Specific Gravity

The density of a substance is a measure of its mass for a given volume. It is usually expressed in units of grams per cubic centimetre ( $\text{g/cm}^3$ ) or kilograms per cubic metre ( $\text{kg/m}^3$ ).

Specific gravity is a measure that compares the density of a substance to another. The basis for comparison for liquids and solids is water which has a density of 1 gram per cubic centimetre.

The specific gravity of a substance will determine whether it will sink (sp gr  $>1$ ) or float (sp gr  $<1$ ) and can therefore be removed through sedimentation or floatation.

The formulas are:

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\text{Specific gravity} = \frac{\text{Mass of the substance, kg/L}}{\text{Mass of 1 litre of water}}$$

### Density

**A substance weighs 321 grams and occupies a volume of 160 mL. What is its density in  $\text{g/cm}^3$ ?**

Insert known values and solve

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{321 \text{ g}}{160 \text{ cm}^3} = 2.0 \text{ g/cm}^3$$

**The density of an unknown substance is  $3.55 \text{ g/cm}^3$ . How much space will 1,445 grams of the substance occupy?**

Insert known values and solve:

$$\text{Volume} = \frac{\text{Mass}}{\text{Density}} = \frac{1,445 \text{ g}}{3.55 \text{ g/cm}^3} = 407 \text{ cm}^3$$

### Specific Gravity of Liquids

**How much will the contents of a 205 L drum full of sodium hypochlorite weigh if the specific gravity of solution is 1.19?**

$$\text{Specific gravity} = \frac{\text{Mass of the substance, kg/L}}{\text{Mass of 1 litre of water}}$$

Step 1 – rearrange the formula to solve for mass.

$$\text{Mass} = \text{Specific gravity} \times 1 \text{ kg/L} \times \text{Volume} = 1.19 \times \frac{1 \text{ kg}}{\text{L}} \times 205 \text{ L} = 244 \text{ kg}$$

**The density of an unknown liquid is 1.32 kilograms per litre. What is the specific gravity of the liquid?**

Insert known values and solve

$$\text{Specific gravity} = \frac{\text{Mass of the substance, kg/L}}{\text{Mass of 1 litre of water}} = \frac{1.32 \text{ kg/L}}{1 \text{ kg/L}} = 1.32$$



### Specific Gravity of Solids

**A piece of metal that weighs 28.4 kilograms in air is weighed in water and found to weigh 19.2 kilograms. What is the specific gravity of this metal?**

Step 1 – Subtract the weight in water from the weight in air to determine the loss of weight in water

$$\text{Weight loss} = 28.4 \text{ kg} - 19.2 \text{ kg} = 9.2 \text{ kg of weight loss in water}$$

Step 2 – Find the specific gravity by dividing the weight of the metal in air by the weight loss in water

$$\text{Specific gravity} = \frac{\text{Weight of the substance in air}}{\text{Loss of weight in water}} = \frac{28.4 \text{ kg}}{9.2 \text{ kg}} = 3.1$$

## Typical Certification Questions for Level 1

- Determine the daily average total concentration of a chemical given the following data:  
Monday 0.0069 mg/L    Tuesday 0.0085 mg/L    Wednesday 0.0094 mg/L  
Thursday 0.0079 mg/L    Friday 0.0052 mg/L
- Find the area of a tank that is 14 metres in diameter
- What is the pressure in kPa at the bottom of a tank if the water is 11.67 metres deep?
- If a trench is 50 metres long, 1.2 metres wide and 1.8 metres deep, how many cubic metres of soil were excavated?
- If exactly 375 litres of polymer cost \$22.50, what will 19,000 litres cost assuming no quantity discount?
- What is the velocity of flow in metres per second for a 250 mm diameter pipe if it delivers 35.5 litres per second?
- What should the setting be on a chlorinator in kilograms per day if the dosage desired is 3 mg/L and the volume of flow is 55 litres per second?
- A treatment plant uses 68 kilograms of chlorine gas per day. If the chlorine demand is 1.85 mg/L and the chlorine residual is 0.2 mg/L, what was the flow in Megalitres per day?
- If 45 kilograms of magnesium hydroxide  $[\text{Mg}(\text{OH})_2]$  is dissolved in 750 litres of water what is the percent strength of the  $\text{Mg}(\text{OH})_2$  solution?
- Calculate the volume in cubic metres of a pipeline 760 mm in diameter and 4.5 kilometres long.
- Given the following data, calculate the solids loading rate on a secondary clarifier:  
Diameter = 30 metres    Flow = 9200 m<sup>3</sup>/day    MLSS = 2,560 mg/L
- Determine the waste activated sludge pumping rate in litres per second given the following data:  
Amount of WAS to be wasted = 1,800 kg/day    WAS concentration = 4,610 mg/L
- A treatment plant with an influent flow of 14,000 m<sup>3</sup>/day has primary influent suspended solids of 225 mg/L. If the primary effluent suspended solids are 98 mg/L how many kilograms of dry solids are produced per day?
- A lagoon receives a flow of 1,200 m<sup>3</sup>/day. What is the organic loading rate in kg of BOD per day per hectare if the pond has a surface area of 3.5 hectares and the influent BOD is 212 mg/L?
- Determine the amount of COD entering an aeration basin in mg/L if the flow is 6,965 m<sup>3</sup>/day and the COD loading is 1,200 kg/day.

16. Given the following data, calculate the Mean Cell Residence Time (MCRT) for this activated sludge system:  
Aeration tank volume = 2,900 m<sup>3</sup>    Secondary clarifier volume = 900 m<sup>3</sup>    MLSS = 2,475mg/L  
WAS = 1,100 kg/day    Effluent suspended solids = 18 kg/day
17. Given the following data, calculate the food to microorganism (F:M) ratio:  
Primary effluent flow = 10,939 m<sup>3</sup>/day    Aeration tank volume = 1200 m<sup>3</sup>  
MLVSS = 2,270 mg/L    BOD = 190 mg/L
18. What is the solids loading for a dissolved air floatation thickener (DAF) in kg/hr/m<sup>2</sup> if the DAF is 19 metres long by 4 metres wide and it receives a WAS flow of 600 m<sup>3</sup>/day at a concentration of 4,200 mg/L?
19. A sludge drying bed is 90 metres long and 15 metres wide. If sludge were applied to the drying bed to a depth of 120 mm, how many litres of sludge were applied?
20. A filter has a surface area of 90 square metres. What is the filtration rate in litres per minute per square metre if it receives a flow of 320 litres per second?

## Typical Certification Questions for Level 2

1. What is the internal surface area of a cylindrical tank (top, bottom, and cylinder wall) if it is 6 metres high and 14 metres in diameter?
2. A trench that averages 1.5 metres wide and 1.8 metres deep is dug for the purpose of installing a pipe that is 600 mm in diameter. If the trench is 355 metres long how many cubic metres of soil will be required for backfill after the pipe is put in place?
3. A tank that is 5.5 metres in diameter and 4.25 metres tall is being filled at a rate of 63.8 L/minute. How many hours will it take to fill the tank?
4. An unknown substance has a density of 2.86 g/cm<sup>3</sup>. How much space will it occupy if it weighs 10.34 kilograms?
5. How many litres of a sodium hypochlorite solution that contains 5% available chlorine are needed to disinfect a 360 mm diameter pipeline that is 282 metres long if the dosage required is 30.0 mg/L?
6. A 600 mm diameter pipeline 500 metres long was disinfected with calcium hypochlorite tablets containing 60.5% available chlorine. What was the chlorine dosage in mg/L if 11 kilograms of calcium hypochlorite was used?
7. A storage tank has a radius of 18 metres and averages 4.35 metres in depth. What is the average detention time for this storage tank if flow through the tank averages 12,000 cubic metres per day?
8. What should the chlorinator setting be in kilograms per day if a flow of 28,200 cubic metres per day of water is dosed at a rate of 2.0 mg/L?
9. A plant is treating 26.3 Megalitres per day. If lime is being added at a rate of 135.5 grams per minute what is the lime usage in kilograms per day?

10. Water is flowing through a grit channel that is 2.80 metres wide at a rate of  $0.9 \text{ m}^3/\text{s}$ . If the velocity is 0.3 metres per second, what is the depth of water in the channel?
11. The level in equalization basin drops 158 cm in exactly 6 hours. If the basin has a diameter of 29 metres and the plant is producing 27.25 Megalitres per day what is the average discharge rate of the discharge pumps in litres per second?
12. A metering pump discharges 217 mL of alum at a speed setting of 52% and a stroke setting of 35%. If the pump speed is increased to 58% and the stroke remains the same, what will the new discharge rate be?
13. What is the percent volatile solids reduction for a digester if the raw biosolids VSS are 64.3% and the VSS of the digested biosolids is 48.1%?
14. Given the following data, how many kg/day of volatile solids are pumped to a digester:  
Pumping rate = 0.4 L/s      Solids content = 6%  
Volatile solids = 59.1%      Specific gravity = 1.03
15. What is the weir overflow rate (WOR) in litres per day per metre if the flow is  $1,427 \text{ m}^3/\text{day}$  and the radius of the clarifier is 21 metres?
16. Given the following data, calculate the amount of volatile solids removed in kg/day:  
Pumping rate = 12.11 L/s      Pump frequency = 24 times/day      Pump duration = 8 minutes/cycle  
Solids = 3.24%      Volatile solids = 62.5%
17. Find the motor power (mkW) for a pump with the following parameters:  
Motor Efficiency = 89.2%      Total head = 35 metres  
Pump efficiency = 77.9%      Flow =  $7,500 \text{ m}^3/\text{day}$
18. What is the organic loading rate for a trickling filter that is 21 metres in diameter and 155 cm deep in kg BOD/day/ $\text{m}^3$  if the primary effluent flow is 12 Megalitres per day and the BOD is 112 mg/L?
19. Given the following data, calculate the mean cell residence time (MCRT):  
Flow =  $6,662 \text{ m}^3/\text{day}$       Aeration tank volume =  $2,070 \text{ m}^3$       Clarifier Volume =  $1,079 \text{ m}^3$   
MLSS = 2,780 mg/L      WAS = 6,970 mg/L      WAS rate =  $73 \text{ m}^3/\text{day}$       Effluent TSS = 16.5 mg/L
20. What is the food to microorganism ratio for a circular aeration tank 15.66 metres in diameter and 4.81 metres deep if the primary influent flow is 10.83 Megalitres per day, the MLVSS is 2,870 mg/L and the primary effluent has a BOD of 296 mg/L?

**Answer key for practice questions**

Level 1 Questions		Level 2 Questions	
1	0.0076 mg/L	1	571.5 m <sup>2</sup>
2	153.9 m <sup>2</sup>	2	858.2 m <sup>3</sup>
3	114.4 kPa	3	26.4 hours
4	108 m <sup>3</sup>	4	3,615.4 cm <sup>3</sup>
5	\$1,140.00	5	17.2 L
6	0.72 m/s	6	47 mg/L
7	14.27 kg/day	7	2.2 hours
8	36.4 ML/day	8	56.4 kg/day
9	5.6%	9	195.1 kg/day
10	2,040 m <sup>3</sup>	10	1.07 m
11	33 kg/m <sup>2</sup> /day	11	48.3 L/s
12	4.5 L/s	12	242 mL
13	1,778 kg	13	48.5%
14	72.7 kg BOD/ha/day	14	1262 kg
15	8,358 kg/day	15	21,640 L/day/m
16	8.4 days	16	2,825 kg/day
17	0.76	17	42.9 kW
18	1.4 kg/m <sup>2</sup> /hour	18	2.5 kg BOD/m <sup>3</sup> /day
19	16,200 L	19	14 days
20	213 L/minute/m <sup>2</sup>	20	1.2

## Appendix 1 – EOCF Formula Sheets

<b>Conversion Factors and Common Abbreviations for Dimensions</b>	
Pi ( $\pi$ ) = 3.14	A = area
1 BTU = 1,055 kiloJoule	B = base
1 ft-lb = 1.358 Joule	C = circumference
1 hectare (ha) = 10,000 m <sup>2</sup>	D = depth
1 horsepower (electric) = 746 Watt = .746 kW	H = height
1 inch of water column = 0.249 kPa	L = length
1 psi = 6.895 kPa	P = perimeter
1 kPa = 0.102 m of water column	W = width
1 metre of water column = 9.805 kPa	d = diameter
1 kW = 1.34 horsepower (electric)	r = radius
<b>Abbreviations used in Formulas</b>	
BHP or bhp = brake horsepower	BOD = biochemical oxygen demand
BOD <sub>E</sub> = BOD of effluent	BOD <sub>I</sub> = BOD of influent
Conc = concentration (as %, decimal or mg/L)	Den = density (g/cm <sup>3</sup> )
DR (dose) = dosing concentration as mg/L	H = head (in feet or metres)
MLSS = mixed liquor suspended solids	MLSS <sub>F</sub> = final MLSS
MLSS <sub>I</sub> = initial MLSS	MLSS% = MLSS expressed as a %
MLVSS = mixed liquor volatile suspended solids	MHP or mhp = motor horsepower
% Chem = percentage of active ingredient	Q = Flow rate
Q <sub>B</sub> = filter backwash rate	Q <sub>INT</sub> = internal recycle flows in plant
Q <sub>w</sub> = sludge wastage flow	RAS = return activated sludge
SG = specific gravity	SSV or SSV <sub>30</sub> = settled sludge volume
TSS = total suspended solids	TSS <sub>E</sub> = total suspended solids in effluent
Vel = velocity	Vol = volume
VAT = volume of aeration tank	VC = volume of clarifier
VOL <sub>w</sub> = volume of water	WHP or whp = water horsepower
WAS = waste activated sludge	

<b>Length</b>		
Circumference of a circle	$\pi \times \text{diameter}$	$C = \pi \times d$ or $C = 2 \times \pi \times r$
Perimeter of a rectangle	$2 \times (\text{length} + \text{width})$	$P = 2 \times (L+W)$
<b>Area</b>		
Area of a circle	$\pi \times \text{radius} \times \text{radius}$	$\text{Area} = \pi r^2$ or $\frac{d^2}{4}$
Area of a rectangle	$\text{Length} \times \text{Width}$	$A = L \times W$
Area of a triangle	$\frac{1}{2} \text{Base} \times \text{Height}$	$\text{Area} = \frac{\text{Base} \times \text{Height}}{2}$
Surface area of a sphere		$\text{Area} = 4 \times \pi \times r^2$ or $\pi d^2$
<b>Volume</b>		
Rectangular tank	$\text{Length} \times \text{Width} \times \text{Height}$	$L \times W \times H$
Cylindrical tank	$\text{Area} \times \text{Height}$	$\pi \times r^2 \times H$
Pipe	$\text{Area} \times \text{Length}$	$\pi \times r^2 \times L$
Cone	$\frac{1}{3} \times \text{base area} \times \text{height}$	$\frac{\pi \times r^2 \times H}{3}$
Lagoon	Average of the top and bottom Areas $\times$ Depth	$\frac{L_T + L_B}{2} \times \frac{W_T + W_B}{2} \times D$
Sphere (e.g. gas bubble)		$\frac{4\pi r^3}{3}$ or $\frac{\pi d^3}{6}$
<b>Rate of Flow (Q)</b>		
Flow in an open channel	$\text{Width} \times \text{depth} \times \text{velocity}$	$Q = W \times D \times V$
Velocity in an open channel	$\text{Width} \times \text{depth} \div \text{flow}$	$V = \frac{W \times D}{Q}$
Flow in a closed channel (pipe)	$\text{Cross-sectional area} \times \text{velocity}$	$Q = \pi \times r^2 \times V$
Velocity in a closed channel	$\text{Cross-sectional area} \div \text{flow}$	$V = \frac{\pi \times r^2}{Q}$
<b>Detention time (DT) or Hydraulic Retention Time (HRT)</b>		
Detention time in a pipe	$\text{Area} \times \text{length} \div \text{flow}$	$DT = \frac{\pi \times r^2 \times L}{Q}$
Detention time in a tank	$\text{Area} \times \text{depth} \div \text{flow}$	$DT = \frac{L \times W \times D}{Q}$

<b>Hydraulic Loading Rate</b>		
Rotating biological contactor	Flow ÷ surface area of disks	$\frac{Q}{2 \times \pi \times r^2 \times \text{disks}}$
Aeration tank (bioreactor)	Flow ÷ tank volume	$\frac{Q}{L \times W \times D}$
Filter flow rate	Forward flow ÷ surface area	$\frac{Q}{L \times W \times D}$ or $\frac{Q}{\pi \times r^2}$
Filter backwash rate	Backwash flow ÷ surface area	$\frac{Q_b}{L \times W \times D}$ or $\frac{Q_b}{\pi \times r^2}$
<b>Hydraulic Overflow Rate</b>		
Weir overflow rate (WOR)	Flow ÷ clarifier weir length	$\frac{Q}{L}$ or $\frac{Q}{\pi \times d}$
Surface overflow rate (SOR)	Flow ÷ clarifier surface area	$\frac{Q}{L \times W}$ or $\frac{Q}{\pi \times r^2}$
<b>Chemical Feed Rate</b>		
Chemical feed rate (L/day)	Rate of chemical addition based on % concentration and density	$\frac{DR \times Q}{\text{decimal Conc} \times \text{Den} \times 1,000}$
Chlorine dosage (2 formulas available)	Chlorine added ÷ volume treated	$\frac{\text{decimal Conc.} \times V}{\text{Vol}_w, \text{m}^3}$ $\frac{\text{weight, kg} \times 1,000}{\text{Vol}_w, \text{m}^3}$
<b>Organic Loading Rate</b>		
TSS to clarifier	Flow x TSS ÷ area of clarifier	$\frac{Q \times \text{TSS}}{L \times W}$ or $\frac{Q \times \text{TSS}}{\pi \times r^2}$
BOD to aeration tank	Kg BOD added ÷ tank volume	$\frac{Q \times \text{BOD}}{L \times W \times D}$ or $\frac{Q \times \text{BOD}}{\pi \times r^2 \times D}$
BOD to RBC	Kg BOD added ÷ disk area	$\frac{Q \times \text{BOD}}{2 \times \pi \times r^2 \times \text{disks}}$
TSS to Filter	Influent TSS ÷ filter surface area	$\frac{Q \times \text{TSS}}{L \times W}$ or $\frac{Q \times \text{TSS}}{\pi \times r^2}$
MLSS to clarifier	Internal flow x MLSS ÷ surface area of clarifier	$\frac{Q \times \text{TSS}}{L \times W}$ or $\frac{Q \times \text{TSS}}{\pi \times r^2}$



<b>Wastewater Sludge Calculations</b>		
Sludge Volume Index (SVI)	Volume occupied by 1 gram of sludge	$\frac{SSV, \%}{MLSS, \%}$
Sludge Density Index (SDI)	$100 \div SVI$	$\frac{100}{SVI}$
Food to Microorganism Ratio	Kg BOD $\div$ kg MLVSS under aeration	$\frac{Q \times BOD}{MLSS \times (V_{AT} + V_C)}$
Sludge Recycle Rate	Fraction of influent flow in the sludge recycle	$Q_R = \frac{Q \times MLSS}{RAS - MLSS}$ or $\frac{Q}{(MLSS, \% \times SVI) - 1}$
Sludge Wasting Rate	Sludge wasted from the process to maintain MLSS	$Q_W = \frac{(MLSS_I - MLSS_F) \times V_{AT}}{RAS}$
Mean Cell Residence Time	Length of time solids remain in the system	$MCRT = \frac{MLSS \times (V_{AT} + V_C)}{(Q \times TSS) + (Q_W \times WAS)}$
<b>Horsepower</b>		
Brake horsepower – Imperial	Horsepower required to drive a pump	$hp = \frac{Q, USgpm \times H, ft \times SG}{3960 \times \text{pump efficiency}}$
Brake horsepower - Metric		$kw = \frac{9.81 \times Q, m^3/s \times H, m \times SG}{\text{pump efficiency}}$
<b>Efficiency</b>		
Efficiency of treatment	Input – output as a percentage of input	$\frac{\text{value in} - \text{value out}}{\text{value in}} \times 100$
Motor efficiency	Motor output energy as a percentage of input energy	$\frac{100 \times bhp}{mhp}$
Pump efficiency	Water output energy as a percentage of motor input energy	$\frac{100 \times whp}{bhp}$
Overall efficiency	Water output energy as a percentage of input electrical energy	$\frac{100 \times whp}{mhp}$

## Appendix 2 – ABC Formula Sheets

$$\text{Alkalinity, as mg CaCO}_3/\text{L} = \frac{\text{titrant volume, mL} \times \text{acid normality} \times 50,000}{\text{sample volume, mL}}$$

$$\text{Amperes (Amps, I)} = \frac{\text{Volts (V or E)}}{\text{Ohms (R)}}$$

$$\text{Area of cone (lateral surface area)} = \pi \times (\text{radius})^2 \times \sqrt{(\text{radius})^2 + (\text{height})^2}$$

$$\text{Area of cone (total surface area)} = \pi \times \text{radius} \times (\text{radius} + \sqrt{(\text{radius})^2 + (\text{height})^2})$$

$$\text{Area of a cylinder (total outside surface area)} = 2 \times \pi \times (\text{radius})^2 + \pi \times \text{diameter} \times \text{height}$$

$$\text{Area of a rectangle or square} = \text{length} \times \text{width}$$

$$\text{Area of a right triangle} = \frac{\text{base} \times \text{height}}{2}$$

$$\text{Area of a sphere} = 4 \times \pi \times (\text{radius})^2$$

$$\text{Average (arithmetic mean)} = \frac{\text{sum of all terms}}{\text{number of terms}}$$

$$\text{Average (geomtric mean)} = [X_1 \times X_2 \times X_3 \times \dots X_n]^{\frac{1}{n}}$$

$$\text{Average (geometeric mean)} = [(X_1)(X_2)(X_3)(X_4)\dots(X_n)]^{1/n} \text{ The } n\text{th root of the product of } n \text{ numbers}$$

Biochemical oxygen demand (BOD)

$$\text{BOD (unseeded)} = \frac{\text{initial D.O., mg/L} - \text{final D.O. mg/L}}{\text{sample volume, mL}} \times \text{total volume, mL}$$

$$\text{BOD (seeded)} = \frac{\text{initial D.O. mg/L} - \text{final D.O. mg/L} - \text{seed correction mg/L}}{\text{sample volume, mL}} \times \text{total volume, mL}$$

$$\text{Seed correction} = \frac{\text{BOD of seed stock, mg/L} \times \text{Volume of seed stock, mL}}{\text{Total volume, mL}}$$

## Metric Mathematics for Operators

### Chemical feed pump setting

$$\% \text{ stroke} = \frac{\text{desired flow} \times 100\%}{\text{maximum flow}}$$

$$\text{mL/minute} = \frac{\text{flow, m}^3/\text{day} \times \text{dose, mg/L}}{\text{chemical feed density, g/cm}^3 \times \% \text{ active chemical} \times 1,440}$$

$$\text{Composite sample single portion} = \frac{\text{instantaneous flow} \times \text{sample volume}}{\text{number of portions} \times \text{average flow}}$$

$$\text{Cycle time, minutes} = \frac{\text{storage volume, m}^3}{\text{pump capacity, m}^3/\text{min} - \text{inflow, m}^3/\text{min}}$$

$$\text{Detention time (hydraulic retention time)} = \frac{\text{Volume}}{\text{Flow}}$$

$$\text{Feedrate, kg/day} = \frac{\text{dosage, mg/L} \times \text{flow rate, m}^3/\text{day}}{\text{chemical purity as a decimal percent} \times 1,000}$$

$$\text{Feedrate, L/min (fluoride saturator)} = \frac{\text{plant capacity, L/min} \times \text{dosage, mg/L}}{18,000 \text{ mg/L}}$$

$$\text{Filter backwash rise rate, cm/min} = \frac{\text{water rise, cm}}{\text{time, minutes}}$$

$$\text{Filter drop test velocity, m/min} = \frac{\text{water drop, m}}{\text{time, minutes}}$$

$$\text{Filter flow rate or backwash rate, L/m}^2/\text{s} = \frac{\text{Flow, L/s}}{\text{Filter area, m}^2}$$

$$\text{Filter yield rate, kg/m}^2/\text{hr} = \frac{\text{Solids concentration, \%} \times \text{sludge feed rate, L/hr} \times 10}{\text{Filter area, m}^2}$$

$$\text{Flow, m}^3/\text{s} = \text{Area, m}^2 \times \text{Velocity, m/s}$$

$$\text{Food to Microorganism ratio (F:M)} = \frac{\text{BOD added, kg/day}}{\text{Mixed liquor volatile solids under aeration, kg}}$$

$$\text{Force, Newtons (N)} = \text{Pressure, Pa} \times \text{Area, m}^2$$

$$\text{Litres/capita/day} = \frac{\text{volume of water produced, L/day}}{\text{population}}$$

## Metric Mathematics for Operators

$$\text{Hardness, as mg CaCO}_3/\text{L} = \frac{\text{titrant volume, mL} \times 1,000}{\text{sample volume, mL}}$$

(only when the titration factor is 1.00 of EDTA)

$$\text{Hydraulic loading rate, m}^3/\text{m}^2/\text{day} = \frac{\text{total flow applied, m}^3/\text{day}}{\text{surface area, m}^2}$$

$$\text{Hypochlorite strength, \%} = \frac{\text{Chlorine required, kg} \times 100}{\text{hypochlorite solution needed, kg}}$$

$$\text{Mass, kg} = \frac{\text{volume, m}^3 \times \text{concentration, mg/L}}{1,000}$$

$$\text{Mean Cell Residence Time (MCRT), days} = \frac{\text{Aeration tank TSS} + \text{Clarifier TSS, kg}}{\text{WAS, kg/day} + \text{Effluent TSS, kg/day}}$$

$$\text{Milliequivalents} = \text{mL} \times \text{Normality}$$

$$\text{Molarity} = \frac{\text{Moles of solute}}{\text{Litres of solution}}$$

$$\text{Normality} = \frac{\text{Number of equivalent weights of solute}}{\text{Litres of solution}}$$

$$\text{Number of equivalent weights} = \frac{\text{Total weight}}{\text{Equivalent weight}}$$

$$\text{Number of moles} = \frac{\text{Total weight}}{\text{Molecular weight}}$$

$$\text{Organic loading rate, kg/m}^3/\text{day} = \frac{\text{Organic load, kg BOD/day}}{\text{Volume, m}^3}$$

$$\text{Organic loading rate - RBC, kg/m}^2/\text{day} = \frac{\text{Organic load, kg BOD/day}}{\text{Media surface area, m}^2}$$

$$\text{Oxygen uptake rate (OUR), mg/L/min} = \frac{\text{Oxygen usage, mg/L}}{\text{Time, minutes}}$$

$$\text{Population equivalent, organic} = \frac{\text{flow, m}^3/\text{day} \times \text{BOD, mg/L}}{1,000 \times 0.77 \text{ kg BOD/person/day}}$$

## Metric Mathematics for Operators

Pump power equations.

Where flow is in m<sup>3</sup>/second use a factor of 9,810 instead of 9.81. Unless the question specifies a different value, assume that the specific gravity (sp.gr) of the fluid being pumped is 1

$$\text{Brake power, Watts} = \frac{9.81 \times \text{flow, L/s} \times \text{head, m} \times \text{sp. gr.}}{\text{decimal pump efficiency}}$$

$$\text{Motor power, Watts} = \frac{9.81 \times \text{flow, L/s} \times \text{head, m} \times \text{sp. gr.}}{\text{decimal pump efficiency} \times \text{decimal motor efficiency}}$$

$$\text{Water power, Watts} = 9.81 \times \text{flow, L/s} \times \text{head, m} \times \text{sp. gr.}$$

$$\text{Water power, kilowatts} = \frac{9.81 \times \text{flow, L/s} \times \text{head, m} \times \text{sp. gr.}}{1,000}$$

$$\text{Recirculation ratio} = \frac{\text{recirculated flow}}{\text{primary influent flow}}$$

Reduction of volatile solids (Van Kleeck formula) all % values expressed in decimal form e.g. 25% = 0.25

$$\text{Reduction of volatile solids, \%} = \frac{(\%VS_{\text{in}} - \%VS_{\text{out}}) \times 100\%}{\%VS_{\text{in}} - (\%VS_{\text{in}} \times \%VS_{\text{out}})}$$

$$\text{Reduction in flow, \%} = \frac{(\text{initial flow} - \text{reduced flow}) \times 100\%}{\text{initial flow}}$$

$$\text{Removal, \%} = \frac{(\text{in} - \text{out}) \times 100\%}{\text{in}}$$

$$\text{Return rate, \%} = \frac{\text{return flow rate} \times 100\%}{\text{influent flow rate}}$$

$$\text{Return sludge rate, solids balance} = \frac{\text{MLSS, mg/L} \times \text{Flow rate}}{\text{RAS, mg/L} - \text{MLSS, mg/L}}$$

$$\text{Slope, \%} = \frac{\text{change in elevation}}{\text{distance}} \times 100$$

$$\text{Sludge density index (SDI)} = \frac{100}{\text{Sludge Volume Index (SVI)}}$$

$$\text{Sludge volume index (SVI)} = \frac{30 \text{ minute settled sludge volume, mL/L} \times 1,000 \text{ mg/g}}{\text{MLSS, mg/L}}$$

## Metric Mathematics for Operators

$$\text{Solids, mg/L} = \frac{\text{weight of dry solids, g} \times 1,000,000}{\text{sample volume, mL}}$$

$$\text{Solids concentration, mg/L} = \frac{\text{weight, mg}}{\text{volume, L}}$$

$$\text{Solids loading rate, kg/m}^2/\text{day} = \frac{\text{solids applied, kg/day}}{\text{surface area, m}^2}$$

$$\text{Specific gravity} = \frac{\text{specific weight of substance, kg/L}}{\text{specific weight of water, kg/L}}$$

$$\text{Solids loading rate, L/m}^2/\text{day} = \frac{\text{flow, L/day}}{\text{surface area, m}^2}$$

$$\text{Three normal equation} = (N_1 \times V_1) + (N_2 \times V_2) = (N_3 \times V_3) \text{ where } V_1 + V_2 = V_3$$

$$\text{Two normal equation} = (N_1 \times V_1) = (N_2 \times V_2) \text{ where } N = \text{concentration (Normality)} \text{ } V = \text{volume}$$

$$\text{Velocity, m/s} = \frac{\text{flow, m}^3/\text{s}}{\text{area, m}^2} \text{ or } \frac{\text{distance, m}}{\text{time, s}}$$

$$\text{Volume of a cone} = \frac{\pi \times (\text{radius})^2 \times \text{height}}{3}$$

$$\text{Volume of a cylinder} = \pi \times (\text{radius})^2 \times \text{height}$$

$$\text{Volume of a rectangular box} = \text{length} \times \text{width} \times \text{depth}$$

$$\text{Watts (DC circuit)} = \text{volts} \times \text{amperes}$$

$$\text{Watts (AC circuit)} = \text{volts} \times \text{amperes} \times \text{power factor}$$

$$\text{Watts (3 phase AC circuit)} = \text{volts} \times \text{amperes} \times \text{power factor} \times 1.732$$

$$\text{Weir overflow rate, L/m/day} = \frac{\text{flow, L/day}}{\text{weir length, m}}$$

$$\text{Wire to water efficiency, \%} = \frac{\text{water power, kW}}{\text{power input, kw} \times \text{motor kw}} \times 100$$